Adaptive Control

Chapter 8: Robust digital control design

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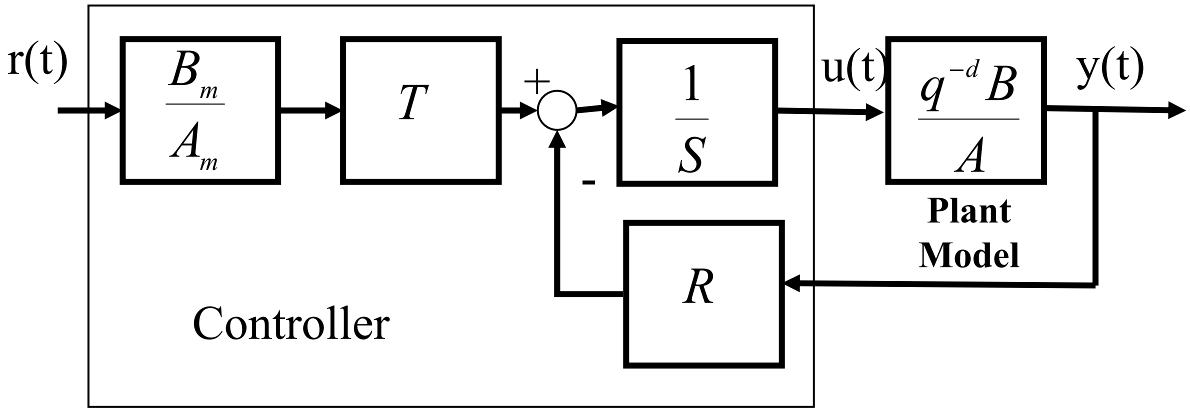
Abstract An adaptive control system has to be built on top of a robust digital control system. Therefore robustness issues for the underlying controller and the shaping of the sensitivity functions for various possible values of the plant parameters are very important. After a review of some basic robustness concepts, a methodology for shaping the sensitivity functions is presented. Its application is illustrated in the context of adaptive control of the flexible transmission.

The R-S-T Digital Controller



Plant Model:

$$\boxed{G(q^{-1}) = H(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} = \frac{q^{-d-1}B^*(q^{-1})}{A(q^{-1})}}$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{n_A} q^{-n_A} \quad B(q^{-1}) = b_1 q^{-1} + \ldots + b_{n_B} q^{-n_B} = q^{-1}B^*(q^{-1})$$

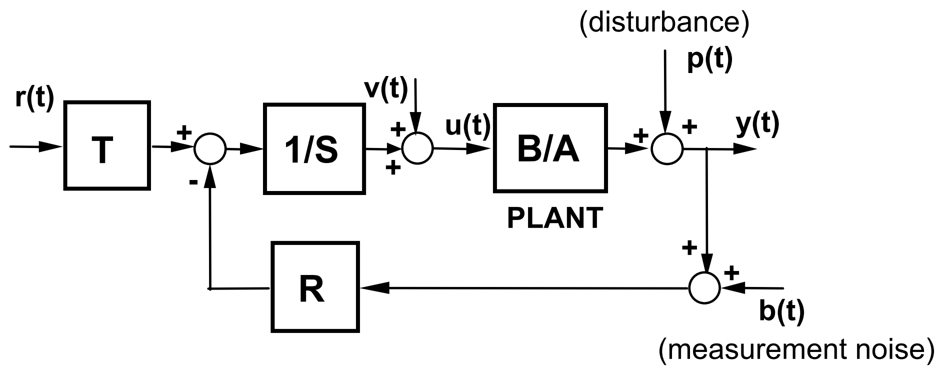
R-S-T Controller:

$$\boxed{S(q^{-1})u(t) = T(q^{-1})y^*(t+d+1) - R(q^{-1})y(t)}$$

Characteristic polynomial (closed loop poles):

$$\boxed{P(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})}$$

Digital control in the presence of disturbances and noise



Output sensitivity function
(p ⟶ y)

$$S_{yp}(z^{-1}) = \frac{A(z^{-1})S(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}$$

Input sensitivity function
(p ⟶ u)

$$S_{up}(z^{-1}) = \frac{-A(z^{-1})R(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}$$

Noise-output sensitivity function
(b ⟶ y)

$$S_{yb}(z^{-1}) = \frac{-B(z^{-1})R(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}$$

Input disturbance-output sensitivity function
(v ⟶ y)

$$S_{yv}(z^{-1}) = \frac{B(z^{-1})S(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}$$

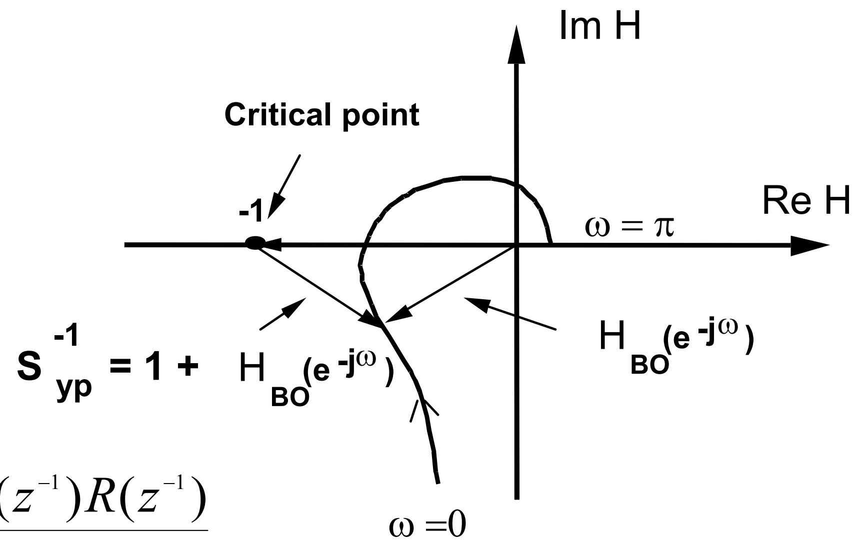
All four sensitivity functions should be stable !

Stability of closed loop discrete time systems

The Nyquist is used like in continuous time

(can be displayed with WinReg ou *Nyquist_OL.sci(.m))*

$$H_{OL}(e^{-j\omega}) = \frac{B(e^{-j\omega})R(e^{-j\omega})}{A(e^{-j\omega})S(e^{-j\omega})}$$



$$S_{yp}^{-1}(z^{-1}) = 1 + H_{OL}(z^{-1}) = \frac{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}{A(z^{-1})S(z^{-1})}$$

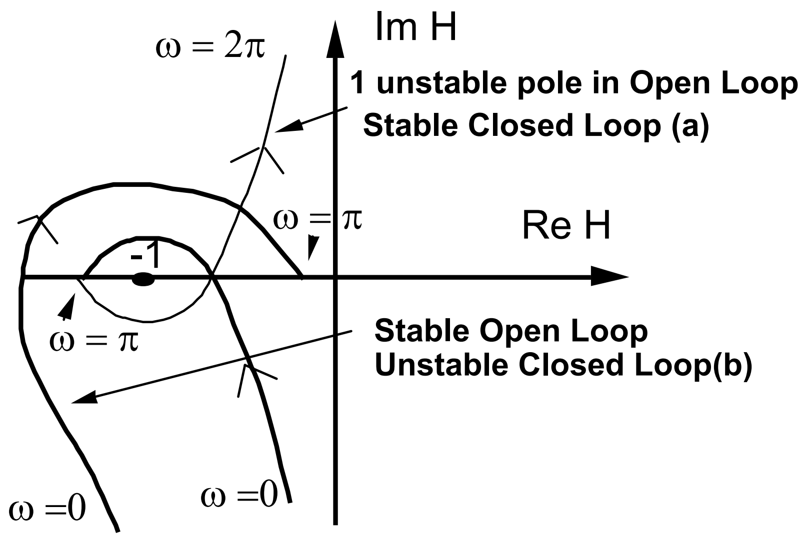
Nyquist criterion (discrete time –O.L. is stable)

The Nyquist plot of the open loop transfer fct. $H_{OL}(e^{-j\omega})$ traversed in the sense of growing frequencies (from 0 to $0.5f_S$) leaves the critical point[-1, j0] on the left

Stability of closed loop discrete time systems

Nyquist criterion (discrete time –O.L. is unstable)

The Nyquist plot of the open loop transfer fct. $H_{OL}(e^{-j\omega})$ traversed in the sense of growing frequencies (from 0 et f_S) leaves the critical point[-1, j0] on the left and the number of encirclements of the critical pointcounter clockwise should be equal to the number of unstable poles in open loop.



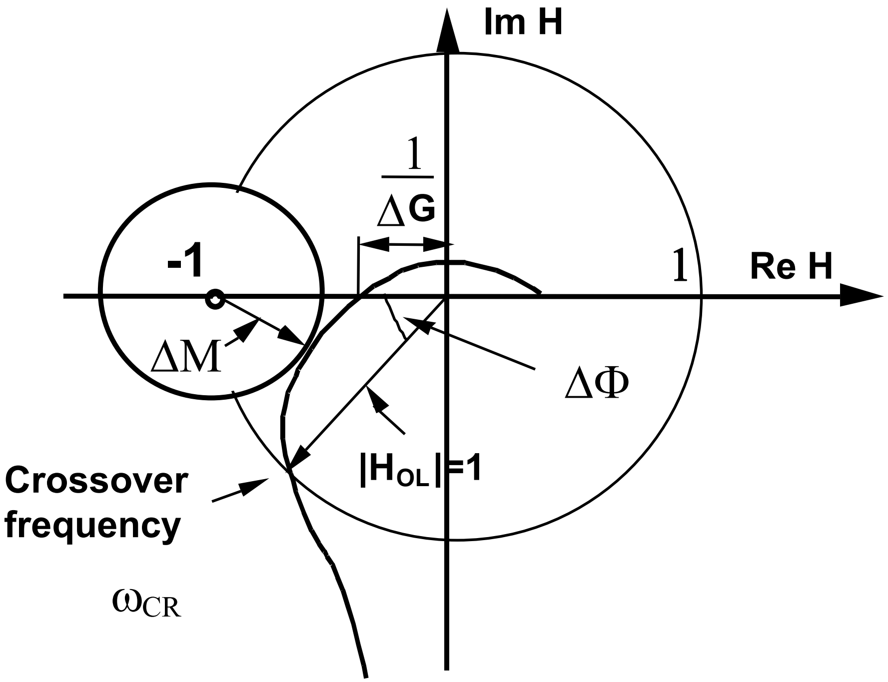
Remarks:

-*The controller poles may become unstable if high performances are required without using an appropriate design method*

-*The Nyquist plot from $0.5f_S$ to f_S is the symmetric with respect to the real axis of the Nyquist plot from 0 to $0.5f_S$*

Marges de robustesse

The minimal distance with respect to the critical point characterizes the robustness of the CL with respect to uncertainties on the plant model parameters(or their variations)



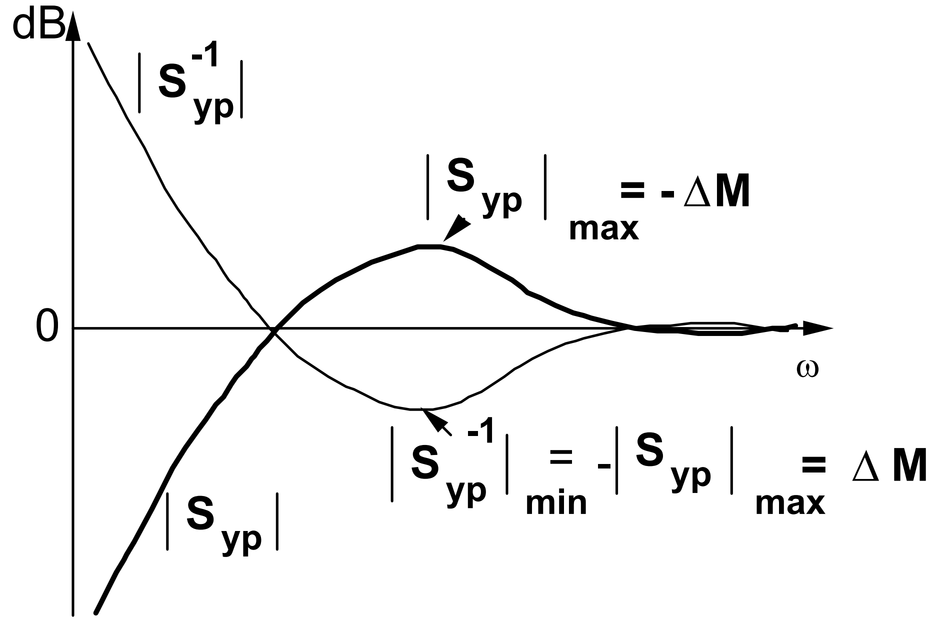
- Gain margin ΔG
- Phase margin $\Delta\phi$
- **Delay margin $\Delta\tau$**
- **Modulus margin ΔM**

Modulus margin and sensitivity function

$$\Delta M = \left|1 + H_{OL}(z^{-1})\right|_{\min} = \left|S_{yp}^{-1}(z^{-1})\right|_{\min} = \left(\left|S_{yp}(z^{-1})\right|_{\max}\right)^{-1} =$$

$$\left(\left|\frac{A(z^{-1})S(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}\right|_{\max}\right)^{-1} \quad pour \quad z^{-1} = e^{-j2\pi f}$$

$$\boxed{\left|S_{yp}(e^{-j\omega})\right|_{\max} dB = \Delta M^{-1} dB = -\Delta M \ dB}$$



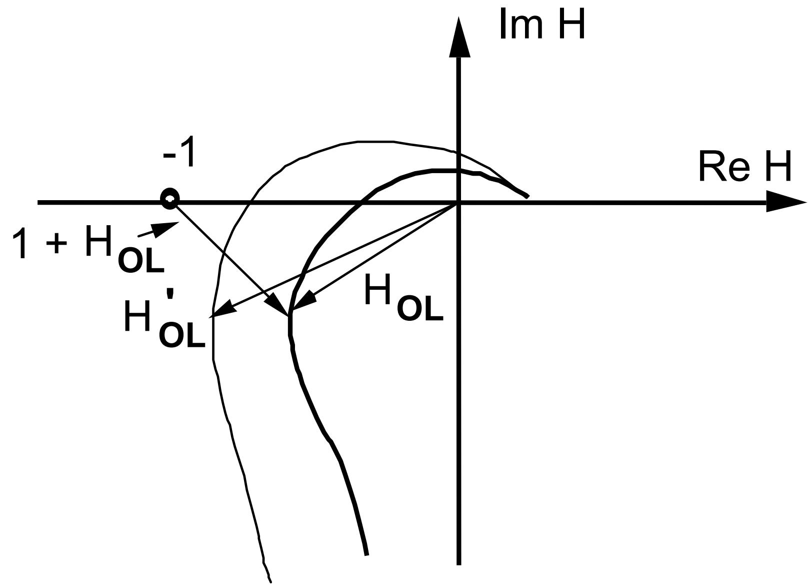
Robust stability

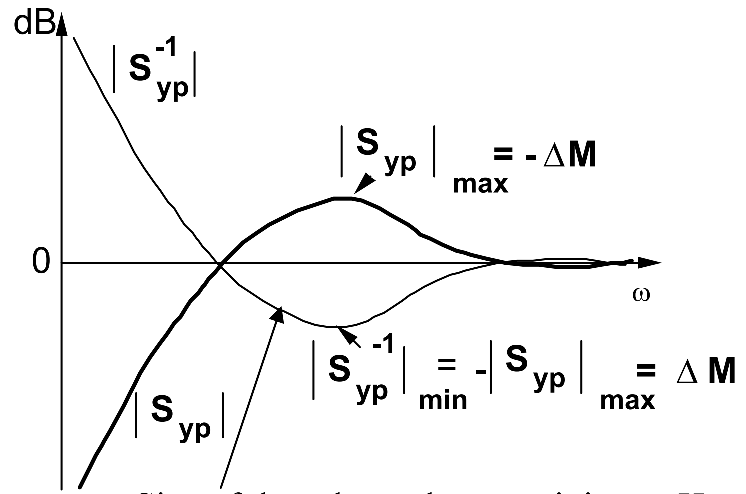
To assure stability in the presence of uncertainties (or variations) on the dynamic chatacteristics of the plant model

H_{OL} – nominal F.T.; H'_{OL} –Different from H_{OL} (perturbed)

Robust stability condition (sufficient cond.):

$$\left|H'_{OL}(z^{-1}) - H_{OL}(z^{-1})\right| < \left|1 + H_{OL}(z^{-1})\right| = \left|S_{yp}^{-1}(z^{-1})\right| =$$
$$\left|\frac{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}{A(z^{-1})S(z^{-1})}\right| = \left|\frac{P(z^{-1})}{A(z^{-1})S(z^{-1})}\right| \quad ; \quad z^{-1} = e^{-j\omega} \qquad (*)$$



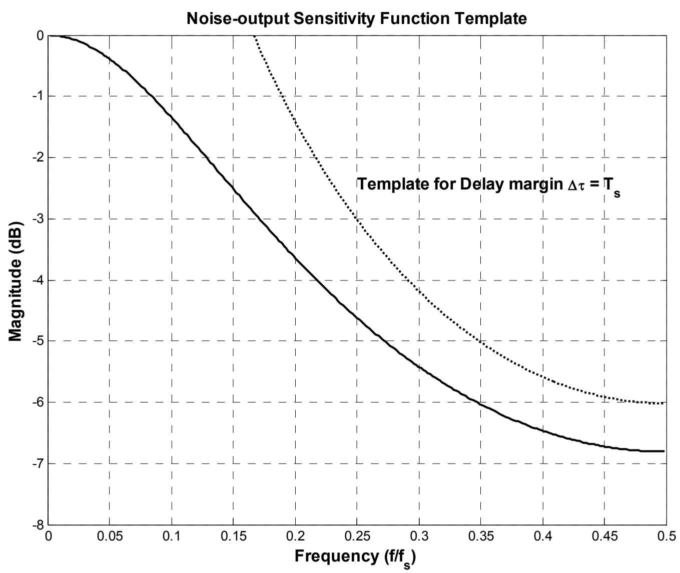


Size of the tolerated uncertainity on H_{OL} at each frequency (radius)

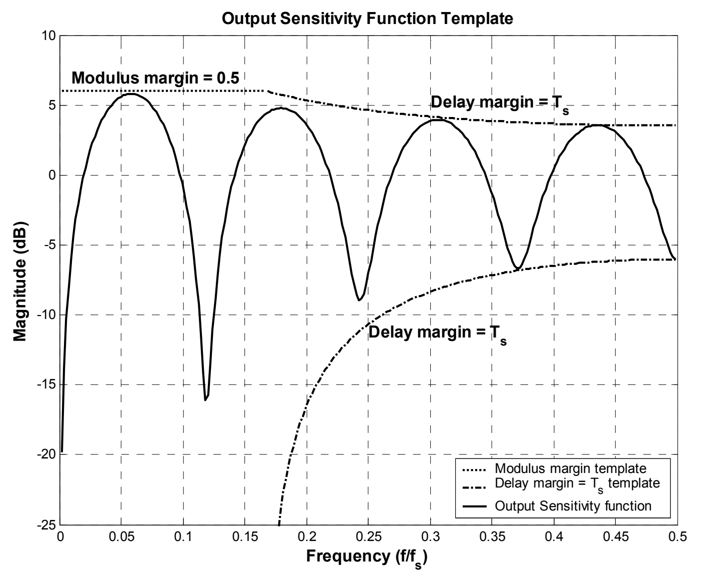
Frequency templates on the sur sensitivity functions

The robust stability conditions allow to define frequency templates on the sensitivity functions which guarantee the delay margin and the modulus margin;

The templates are essential for designing a good controller



Frequency template on the noise-output sensitivity function S_{yb} for $\Delta\tau = T_S$



Frequency template on the output sensitivity function S_{yp} for $\Delta\tau = T_S$ and $\Delta M = 0.5$

Robustness margins

Gain margin

$$\Delta G = \frac{1}{\left|H_{OL}(j\omega_{180})\right|} \qquad \text{pour} \qquad \angle\phi(\omega_{180}) = -180^{\circ}$$

Phase margin

$$\Delta\phi = 180^{0} - \angle\phi(\omega_{cr}) \quad pour \quad \left|H_{BO}(j\omega_{cr})\right| = 1$$

$$\Delta\phi = \min_i \Delta\phi_i$$ If there are several intersections with the unit circle

Delay margin

$$\Delta\tau = \frac{\Delta\phi}{\omega_{cr}}$$

Several intersections points: $\Delta\tau = \min_i \frac{\Delta\phi_i}{\omega_{cr}^{i}}$

Modulus margin

$$\Delta M = \left|1 + H_{OL}(j\omega)\right|_{\min} = \left|S_{yp}^{-1}(j\omega)\right|_{\min} = \left(\left|S_{yp}(j\omega)\right|_{\max}\right)^{-1}$$

Robustness margins – typical values

Gain margin : $\Delta G \geq 2$ $(6\ dB)$ $[min : 1{,}6\ (4\ dB)]$

Phase margin : $30° \leq \Delta\phi \leq 60°$

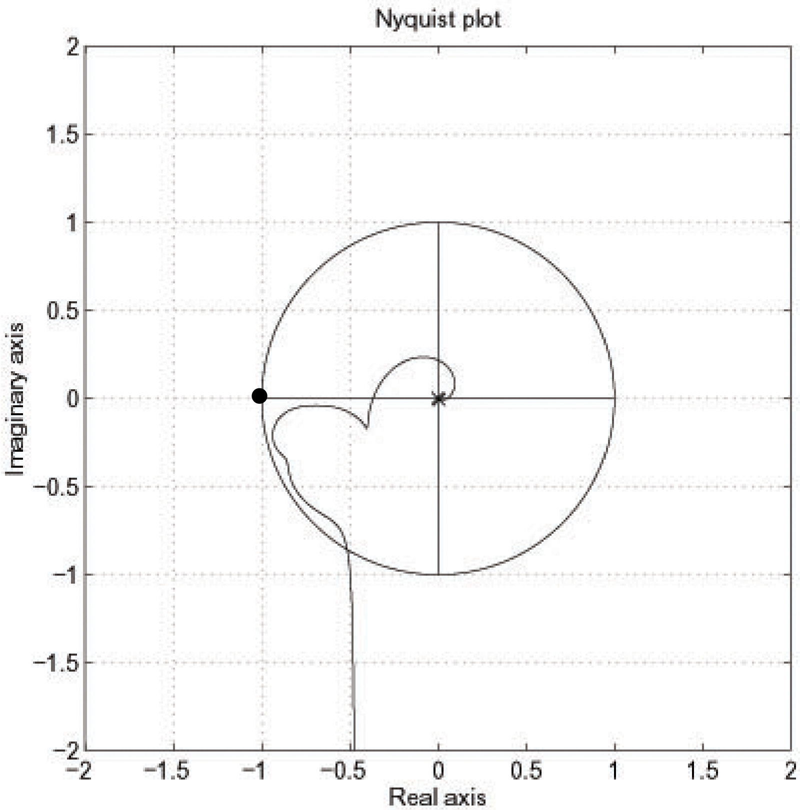
Delay margin : fraction of system delay (10%) or
of time response (10%) (often $1.T_S$)

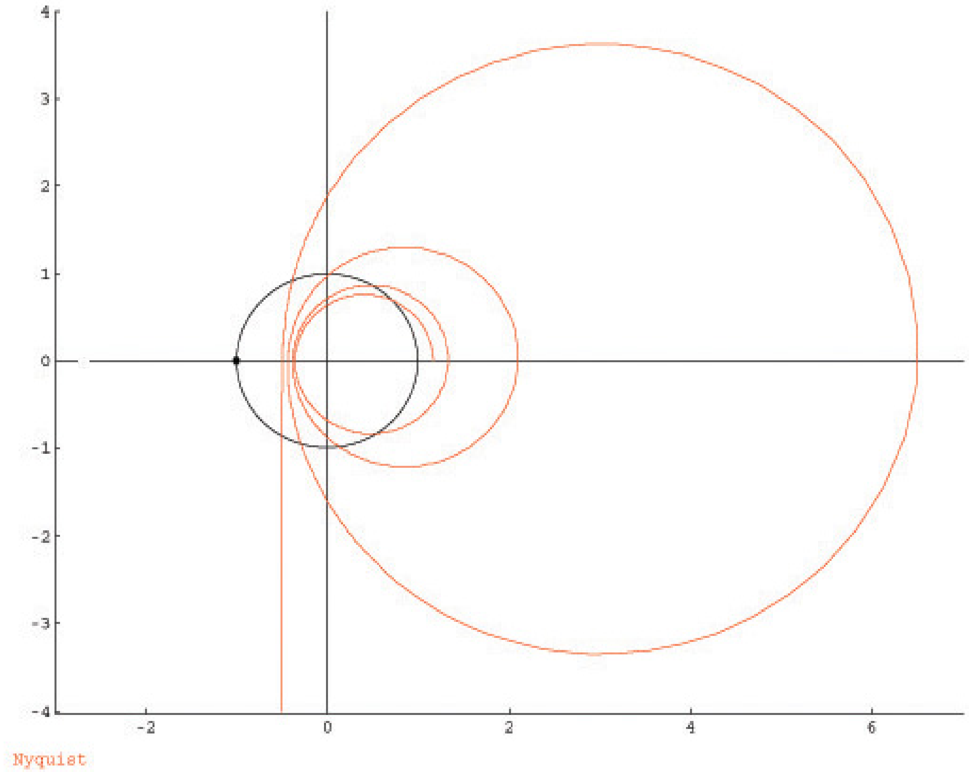
Modulus margin : $\Delta M \geq 0.5$ $(-\ 6\ dB)$ $[min : 0{,}4\ (-8\ dB)]$

A modulus margin $\Delta M \geq 0.5$ implies $\Delta G \geq 2$ *et* $\Delta\phi > 29°$
Attention ! The converse is not generally true

The *modulus margin* defines also the tolerance with respect to nonlinearities

Robustness margins





Good gain and phase margin
Bad modulus margin

Good gain and phase margin
Bad delay margin

Modulus margin and sensitivity function

$$\Delta M = \left|1 + H_{OL}(z^{-1})\right|_{\min} = \left|S_{yp}^{-1}(z^{-1})\right|_{\min} = \left(\left|S_{yp}(z^{-1})\right|_{\max}\right)^{-1} =$$

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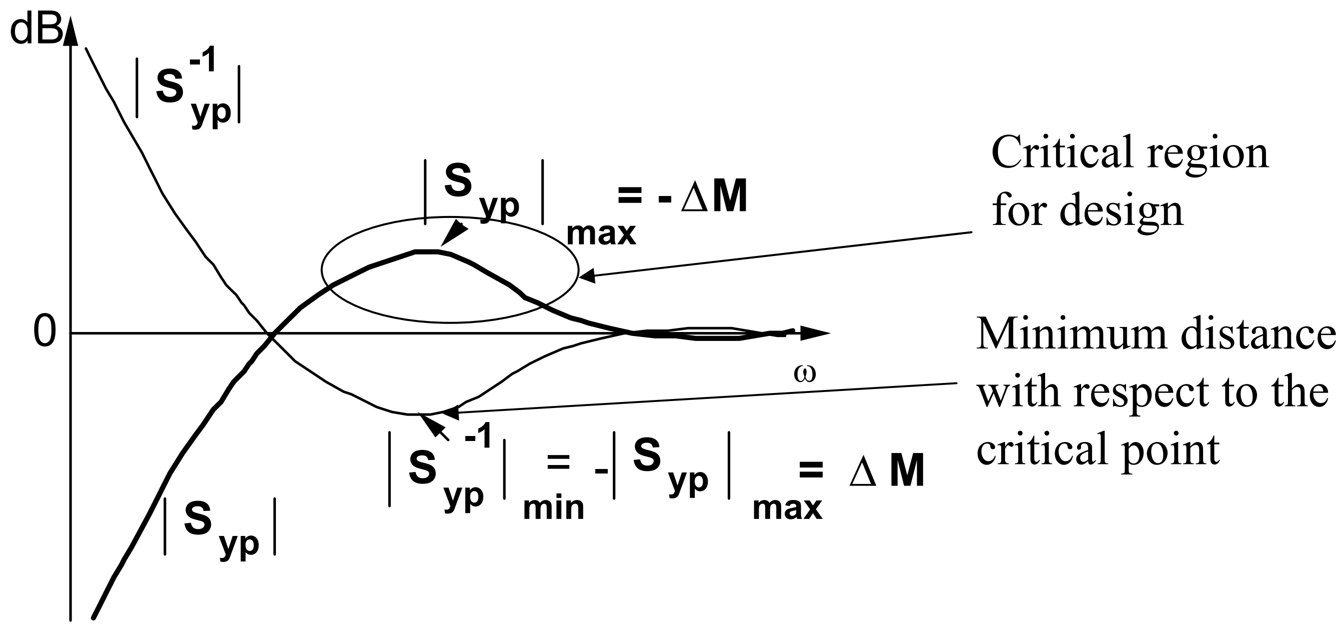
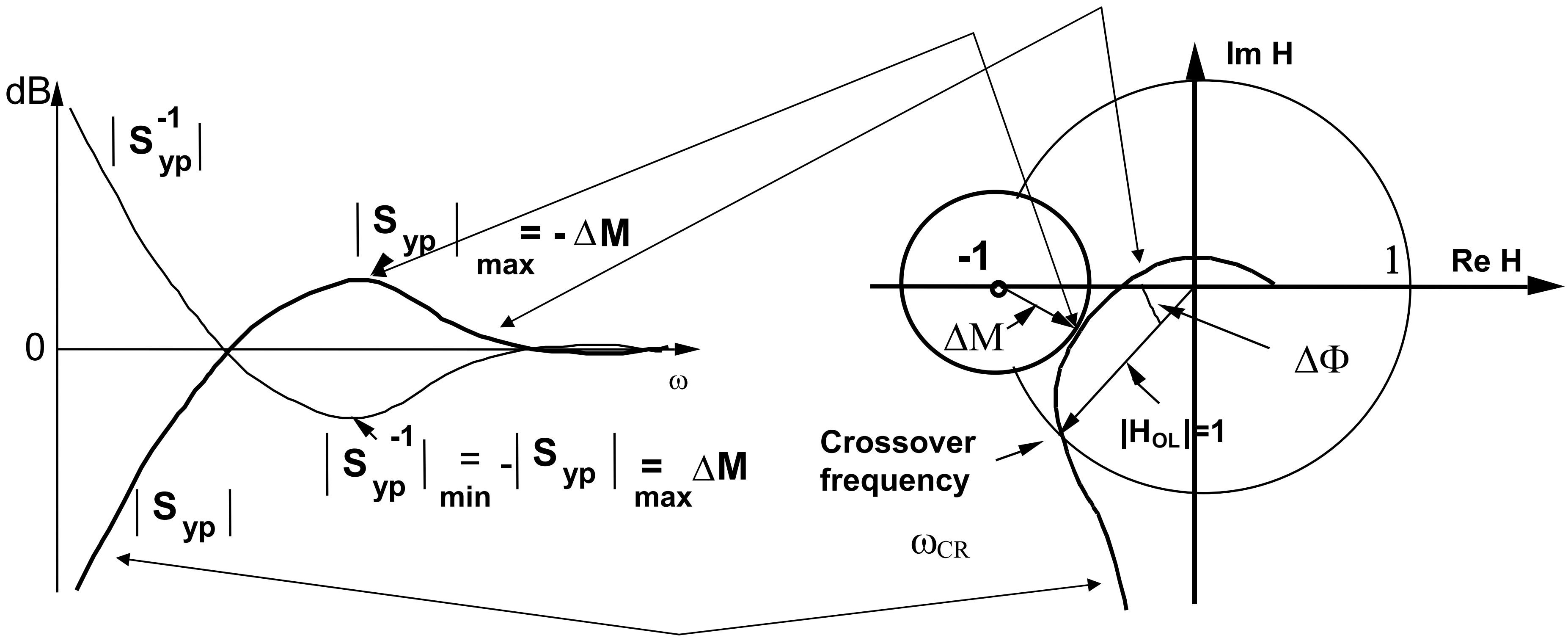


Correspondance Output Sensitivity → Nyquist Plot



Properties of the output sensitivity function

– *The open loop being stable, one has the property:*

$$\int_{0}^{0.5 f_S} \log\left|S_{yp}(e^{-j2\pi f/f_S})\right| df = 0$$

The sum of the areas between the curve of Syp and the axis 0dB taken with their sign is null

$\downarrow$

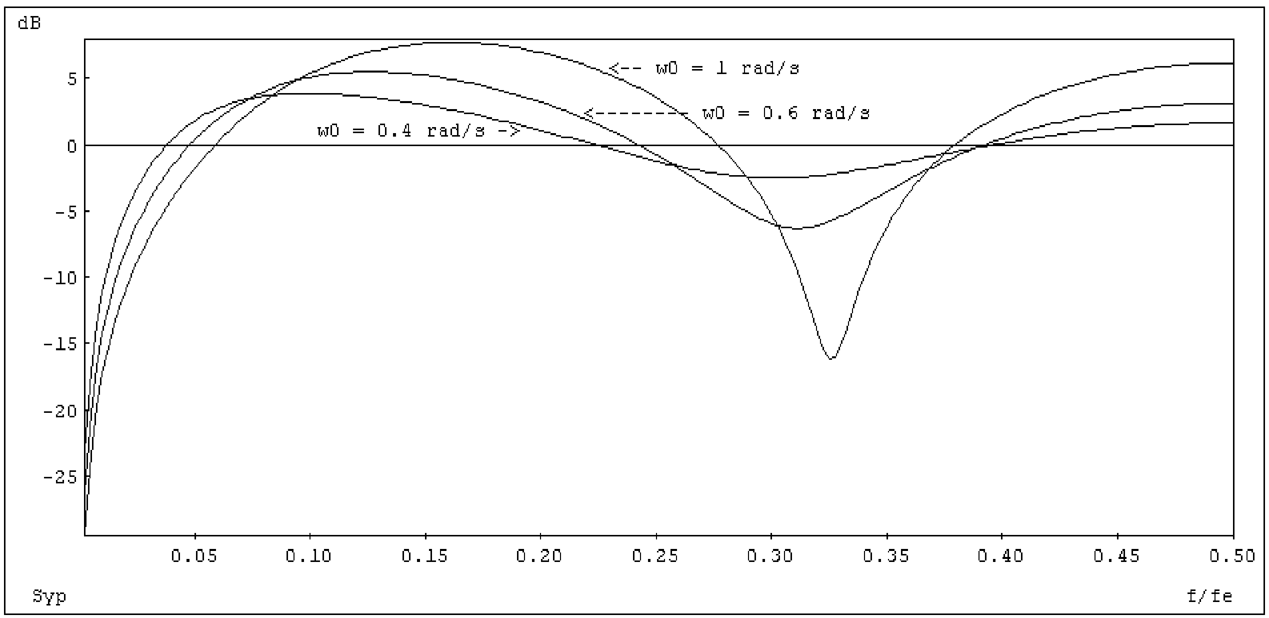
Disturbance attenuation in a frequency region implies amplification of the disturbances in other frequency regions!

Properties of the output sensitivity function

Augmenting the attenuation or widening the attenuation zone

Higher amplification of disturbances ouside the attenuation zone

Reduction of the robustness (reduction of the modulus margin)



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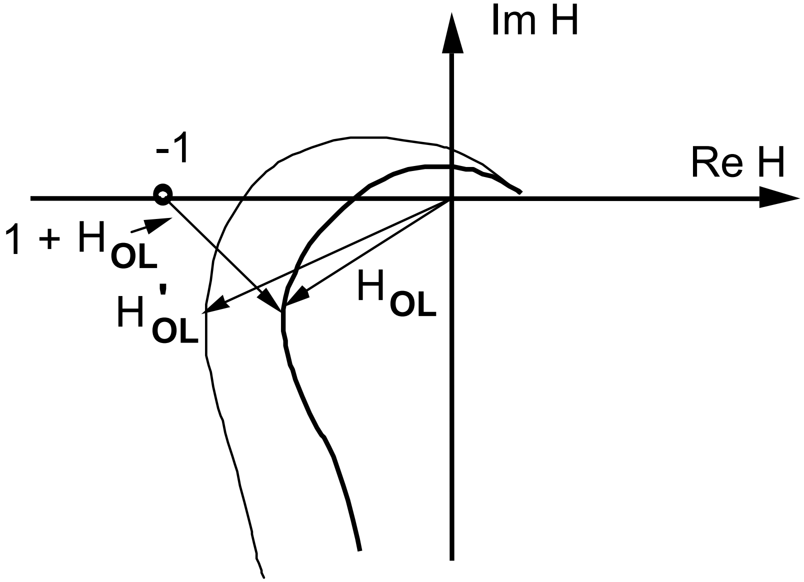
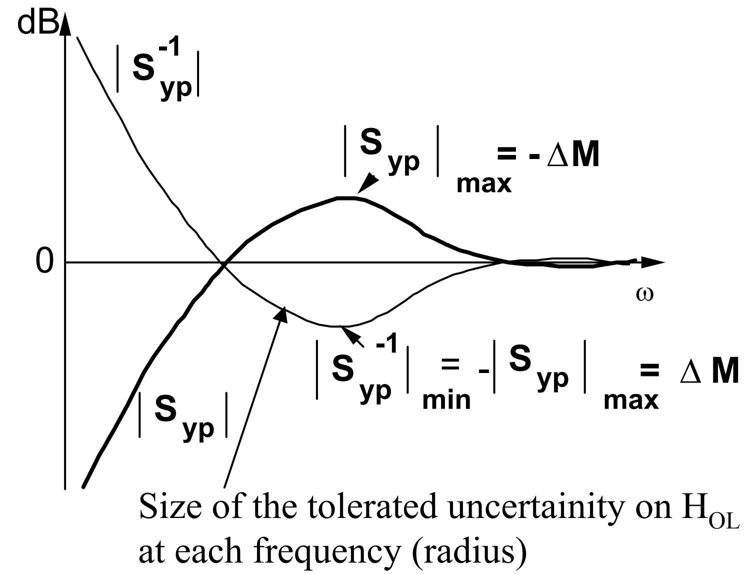
$$\left|\frac{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}{A(z^{-1})S(z^{-1})}\right| = \left|\frac{P(z^{-1})}{A(z^{-1})S(z^{-1})}\right| \quad ; \quad z^{-1} = e^{-j\omega} \qquad (*)$$
$$\left|S_{yp}(z^{-1})\right| < \left|H'_{OL}(z^{-1}) - H_{OL}(z^{-1})\right|^{-1}$$





Tolerance to plant additive uncertainty

From previous slide :

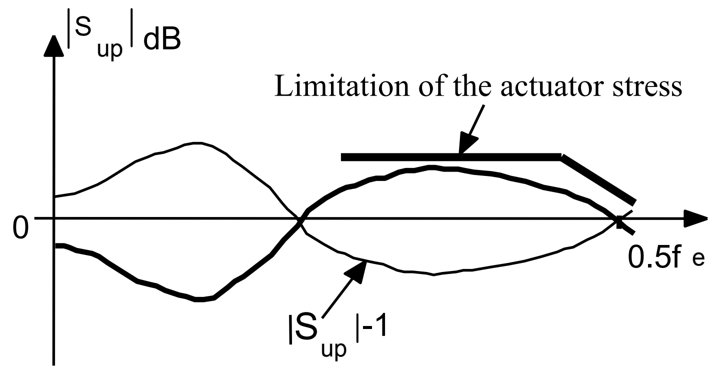
$$\left|\frac{B'(z^{-1})R(z^{-1})}{A'(z^{-1})S(z^{-1})}-\frac{B(z^{-1})R(z^{-1})}{A(z^{-1})S(z^{-1})}\right|=\left|\frac{R(z^{-1})}{S(z^{-1})}\right|\cdot\left|\frac{B'(z^{-1})}{A'(z^{-1})}-\frac{B(z^{-1})}{A(z^{-1})}\right|<\left|\frac{A(z^{-1})S(z^{-1})+B(z^{-1})R(z^{-1})}{A(z^{-1})S(z^{-1})}\right| \quad (*)$$

H'$_{OL}$ H$_{OL}$ G'   G

$$\left|\frac{B'(z^{-1})}{A'(z^{-1})}-\frac{B(z^{-1})}{A(z^{-1})}\right|<\left|\frac{A(z^{-1})S(z^{-1})+B(z^{-1})R(z^{-1})}{A(z^{-1})R(z^{-1})}\right|=\left|\frac{P(z^{-1})}{A(z^{-1})R(z^{-1})}\right|=\left|S_{up}^{-1}(z^{-1})\right| \quad (**)$$

G'   G

$$\boxed{\left|S_{up}(z^{-1})\right|<\left|G'(z^{-1})-G(z^{-1})\right|^{-1}}$$



(Size of tolerated uncertainties)

Tolerance to plant normalized uncertainty (multiplicative uncertainty)

From (**), previous slide:

$$\frac{\left|\frac{B'(z^{-1})}{A'(z^{-1})}-\frac{B(z^{-1})}{A(z^{-1})}\right|}{\left|\frac{B(z^{-1})}{A(z^{-1})}\right|}<\left|\frac{A(z^{-1})S(z^{-1})+B(z^{-1})R(z^{-1})}{B(z^{-1})R(z^{-1})}\right|=\left|\frac{P(z^{-1})}{B(z^{-1})R(z^{-1})}\right|=\left|S_{yb}^{-1}(z^{-1})\right|$$

The inverse of the modulus of the "complementary sensitivity function" gives at each frequency the tolerance with respect to "normalized (multiplicative) uncertainty"

Relation between additive and multiplicative uncertainty:

$$G' = G + (G' - G) = G\left(1 + \frac{G' - G}{G}\right)$$

Important message

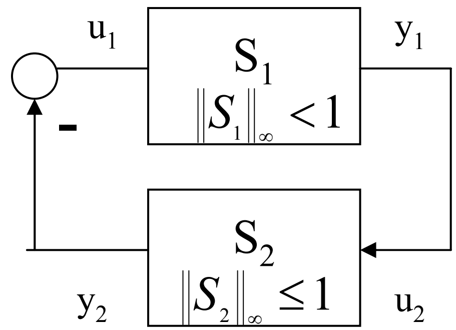
Large values of the modulus of the sensitivity functions in a certain frequency region

→ Low tolerance to model uncertainty

↓

Critical regions for control design
Need for a good model in these regions

Small gain theorem



S_1: linear time invariant (state x)

$$\|S_1\|_\infty < 1$$

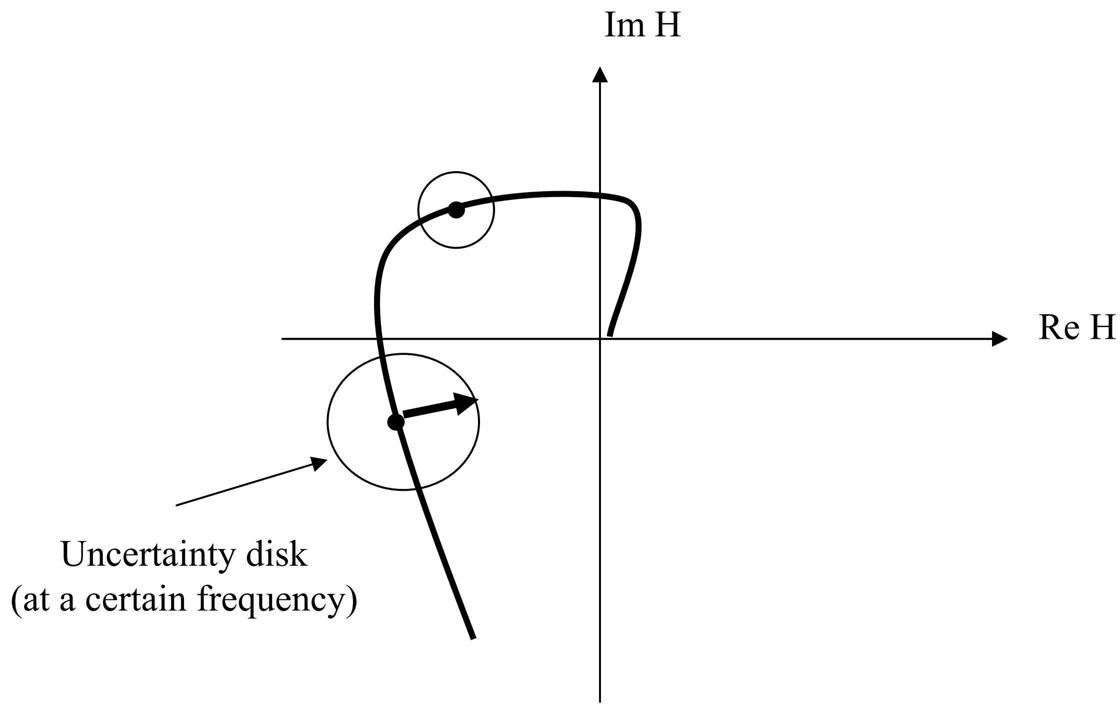
S_2: $\|S_2\|_\infty \le 1$

Then:

$$\lim_{t\to\infty} x(t) = 0\,;\ \lim_{t\to\infty} u_1(t) = 0\,;\ \lim_{t\to\infty} y_1(t) = 0$$

It will be used to characterize "robust stability"

Description of uncertainties in the frequency domain



1) It needs a description by a transfer function which may have any phase but a modulus < 1
2) The size of the radius will vary with the frequency and is characterized by a transfer function

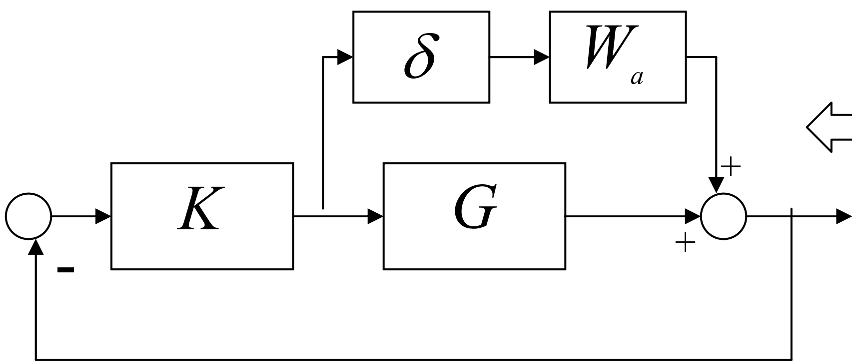
Additive uncertainty

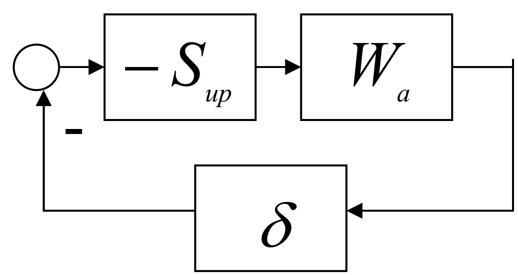
$$G'(z^{-1}) = G(z^{-1}) + \delta(z^{-1})W_a(z^{-1})$$

$\delta(z^{-1})$ any stable transfer function with $\|\delta(z^{-1})\|_\infty \le 1$

$W_a(z^{-1})$ a stable transfer function

$$\left|G'(z^{-1}) - G(z^{-1})\right|_{\max} = \left\|G'(z^{-1}) - G(z^{-1})\right\|_\infty = \left\|W_a(z^{-1})\right\|_\infty$$





Apply small gain theorem

$K = R/S;\ H = z^{-d}B/A$

Robust stability condition: $\left\|S_{up}(z^{-1})W_a(z^{-1})\right\|_\infty < 1$

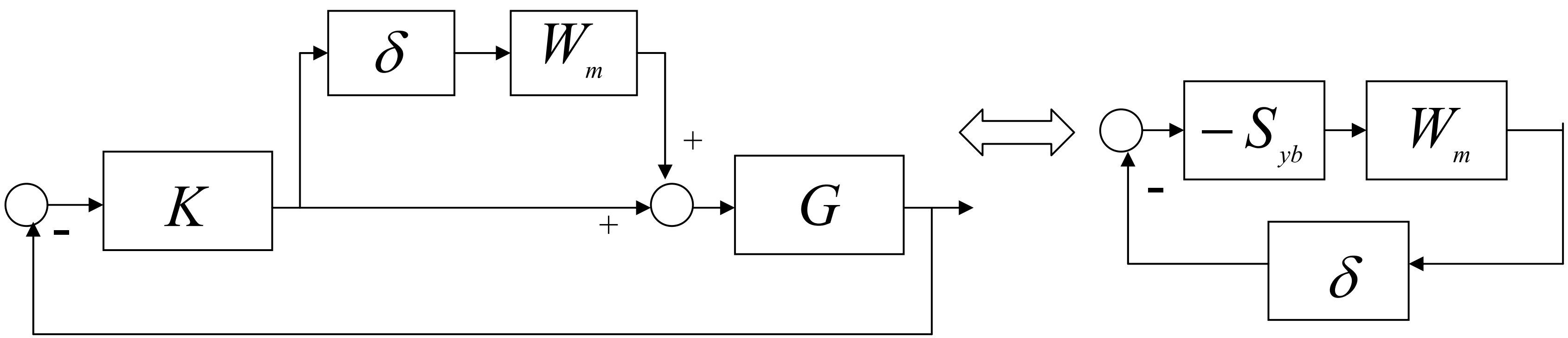
Multiplicative uncertainties

$$G'(z^{-1}) = G(z^{-1})\left[1 + \delta(z^{-1})W_m(z^{-1})\right]$$

$\delta(z^{-1})$ any stable transfer function with $\left\|\delta(z^{-1})\right\|_\infty \leq 1$

$W_m(z^{-1})$ a stable transfer function

$$W_a(z^{-1}) = H(z^{-1})W_m(z^{-1})$$



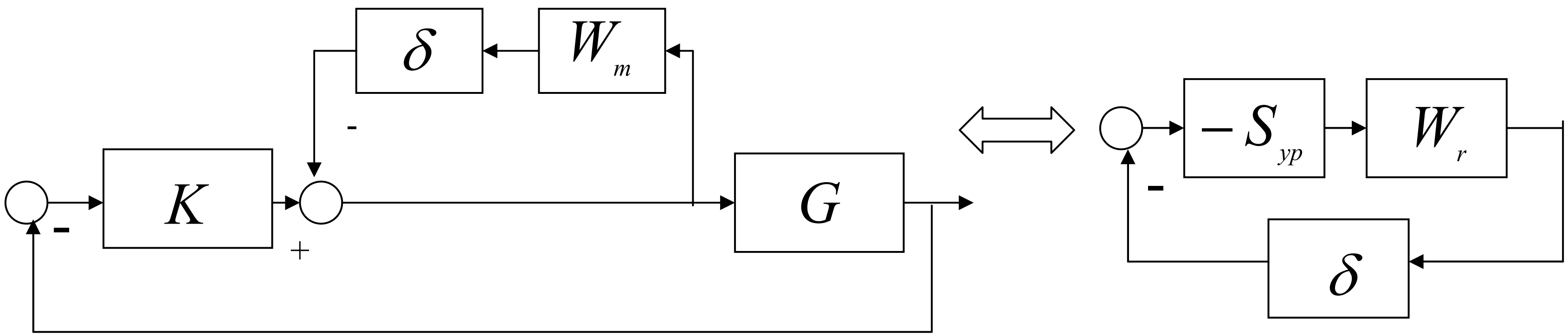
Robust stability condition: $\left\|S_{yb}(z^{-1})W_m(z^{-1})\right\|_\infty < 1$

Feedback uncertainties on the input

$$G'(z^{-1}) = \frac{G(z^{-1})}{\left[1+\delta(z^{-1})W_r(z^{-1})\right]}$$

$\delta(z^{-1})$ any stable transfer function with $\left\|\delta(z^{-1})\right\|_\infty \le 1$

$W_r(z^{-1})$ a stable transfer function



Robust stability condition: $\left\|S_{yp}(z^{-1})W_r(z^{-1})\right\|_\infty < 1$

Robust stability conditions

$H, H' \in \mathrm{P}(W, \delta)$ ⟵ Family (set) of plant models

Robust stability :

The feedback system is asymptotically stable for all the plant models belonging to the family $\mathrm{P}(W, \delta)$

- Additive uncertainties

$$\left\| S_{up}(z^{-1}) W_a(z^{-1}) \right\|_\infty < 1 \quad \Longleftrightarrow \quad \left| S_{up}(e^{-j\omega}) \right| < \left| W_a(e^{-j\omega}) \right|^{-1} \quad 0 \le \omega \le \pi$$

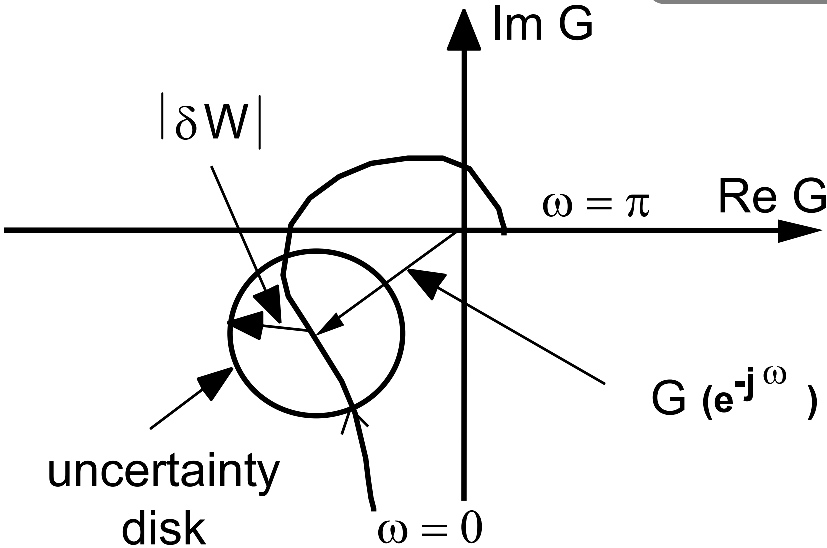
- Multiplicative uncertainties

$$\left\| S_{yb}(z^{-1}) W_m(z^{-1}) \right\|_\infty < 1 \quad \Longleftrightarrow \quad \left| S_{yb}(e^{-j\omega}) \right| < \left| W_m(e^{-j\omega}) \right|^{-1} \quad 0 \le \omega \le \pi$$

- Feedback uncertainties on the input (or output)

$$\left\| S_{yp}(z^{-1}) W_r(z^{-1}) \right\|_\infty < 1 \quad \Longleftrightarrow \quad \left| S_{yp}(e^{-j\omega}) \right| < \left| W_r(e^{-j\omega}) \right|^{-1} \quad 0 \le \omega \le \pi$$

Robust Stability



Family of plant models:

$$G' \in F(G, \delta, W_{xy})$$

$$G - \textit{nominal model;}\ \|\delta(z^{-1})\|_\infty \leq 1$$

$$W_{xy}(z^{-1})\text{- } \textit{size of uncertainty}$$

Robust stability condition:

a related sensitivity function

a type of uncertainty

$$\|S_{xy} W_{xy}\|_\infty < 1$$

$$\Downarrow$$

defines the size of the tolerated uncertainty

$$\boxed{|S_{xy}| < |W_{xy}|^{-1}}$$

defines an upper template for the modulus of the sensitivity function

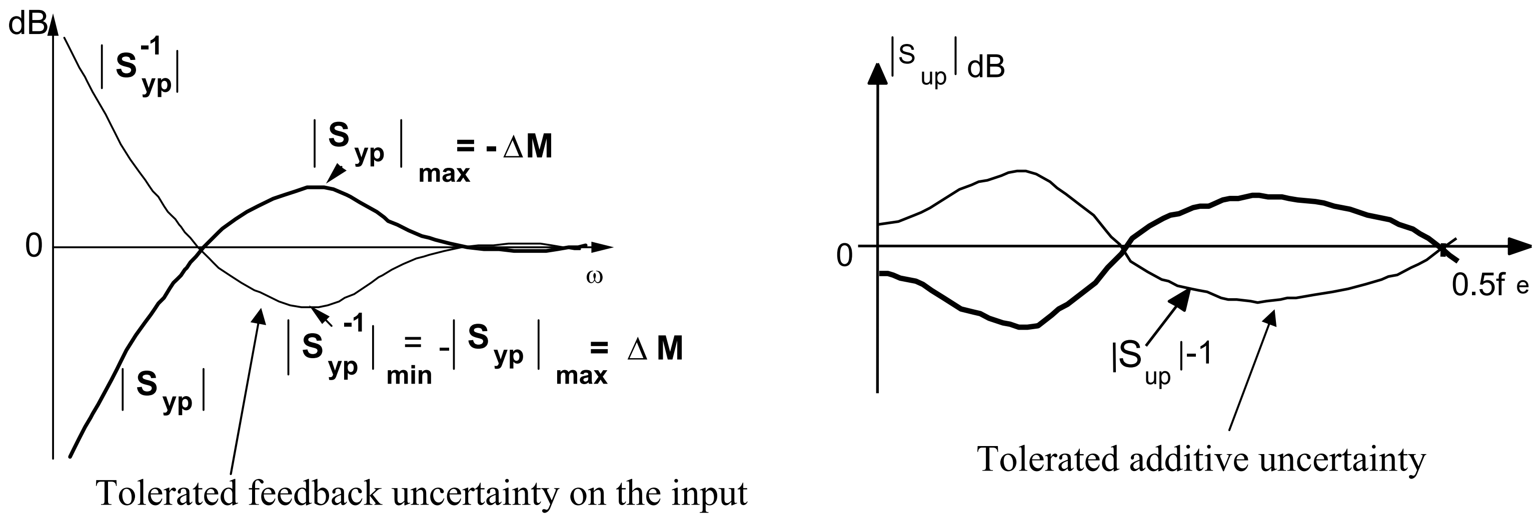
There also lower templates (because of the relationship between various sensitivity fct.)

Robust stability and templates for the sensitivity functions

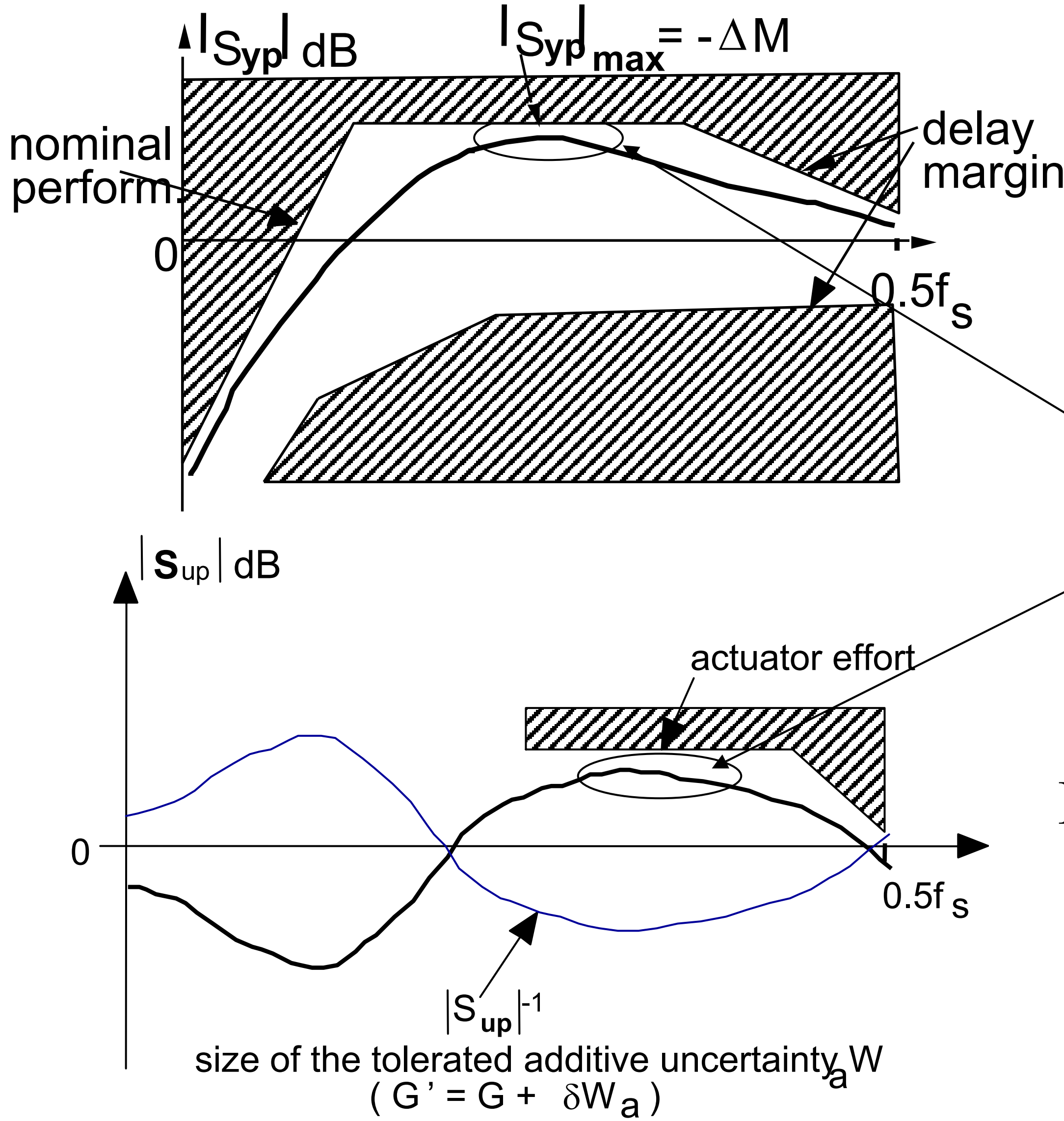
Robust stability condition:

$$\left|S_{xy}(e^{-j\omega})\right| < \left|W_z(e^{-j\omega})\right|^{-1} \quad 0 \le \omega \le \pi$$

- *The functions $\left|W(z^{-1})\right|^{-1}$ (the inverse of the size of the uncertainties) define an "upper" template for the sensitivity functions*
- *Conversely the frequency profile of $\left|S_{xy}(e^{-j\omega})\right|$ can be interpreted in terms of tolerated uncertainties*



Templates for the Sensitivity Functions

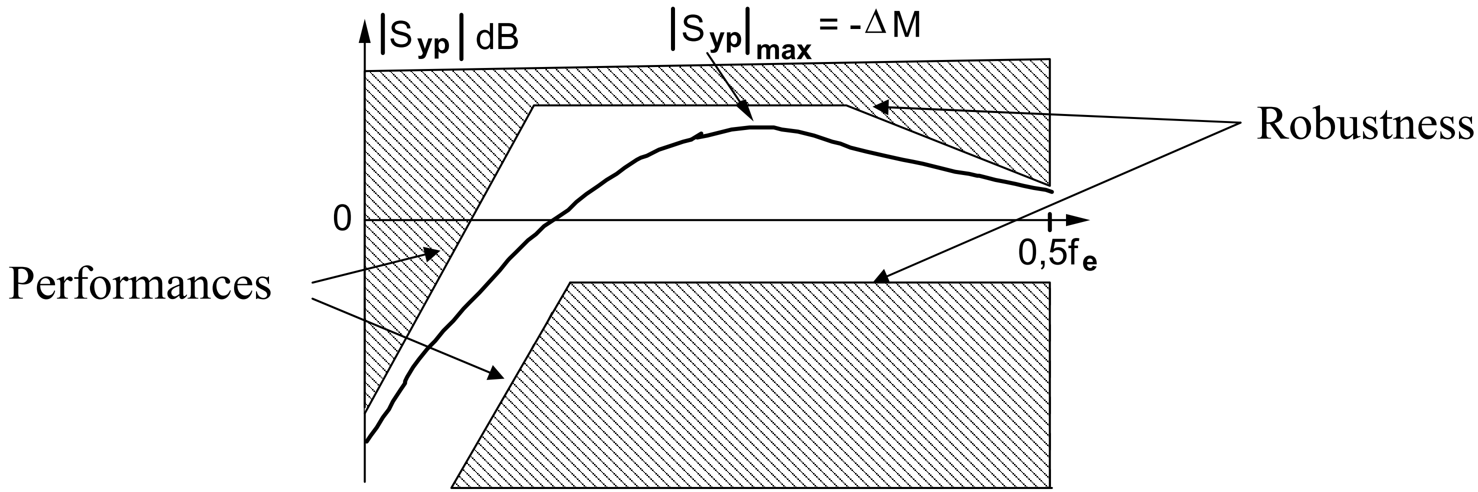


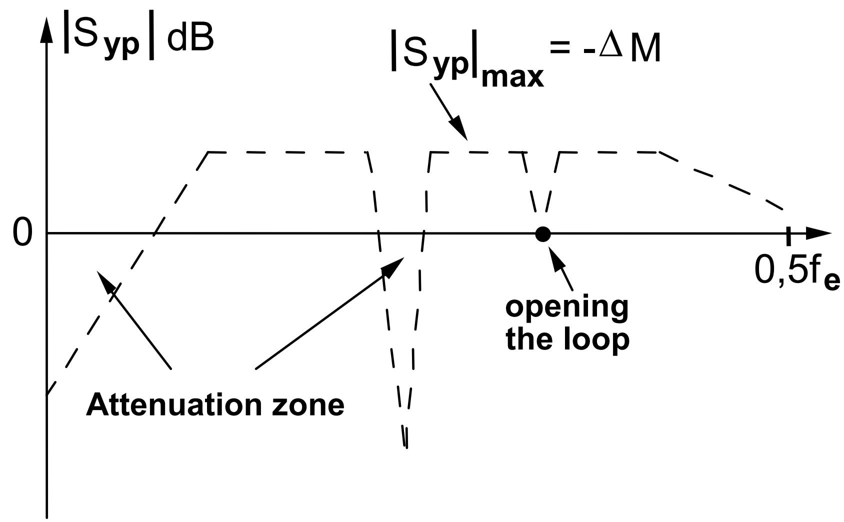
Output Sensitivity Function

Dangerous zones.
Need for good models in these regions

Input Sensitivity Function

Templates for the output sensitivity functions S_{yp}





Shaping the sensitivity functions

1. Choice of the dominants et auxiliary poles of the closed loop
2. Choice of the fixed part of the controller (H_S and H_R)
3. Simultaneous choice of the fixed parts and the auxiliary poles

Procedure:

Basic shaping : use 1 and 2
Fine shaping: use 3

Tools for sensitivity shaping: WinReg (Adaptech) and *ppmaster.m*

There exist also tools for automatic sensitivity function shaping based on convex optimization (Optreg from Adaptech)

Pole placement with sensitivity functions shaping

Performance specification for pole placement :

- Desired dominant poles for the closed loop
- The reference trajectory (tracking reference model)

Questions:

- How to take into account the specifications in certain frequency regions?
- How to guarantee the *robustness* of the controllers ?
- How to take advantage from the degree of freedom for the maximum number of poles which can be assigned ?

Answer:

Shaping the sensitivity functions by:

- introducing auxiliary poles

- introducing filters in the controllers

Sensitivity functions - review

Output sensitivity function:

$$S_{yp}(q^{-1}) = \frac{A(q^{-1})S(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})}$$

Input sensitivity function:

$$S_{up}(q^{-1}) = -\frac{A(q^{-1})R(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})}$$

Controller structure :

$$R(q^{-1}) = R'(q^{-1})H_R(q^{-1}) \qquad S(q^{-1}) = S'(q^{-1})H_S(q^{-1})$$

Pre specified parts (filters)

Dominant and auxiliary filters:

$$A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) = P(q^{-1}) = P_D(q^{-1})P_F(q^{-1})$$

Study of the properties of the sensitivity functions in the frequency domain: $q=z=e^{j\omega}$

Properties of the output sensitivity function

P.1- *The modulus of the output sensitivity function at a certain frequency gives the amplification or attenuation factor of the disturbance on the output*

$S_{yp}(\omega) < 1 (0\ dB)$ attenuation

$S_{yp}(\omega) > 1$ amplification

$S_{yp}(\omega) = 1$ operation in open loop

P.2

$$\Delta M = \left(\left| S_{yp}(j\omega) \right|_{\max} \right)^{-1}$$

Modulus margin

Properties of the output sensitivity function

P.3 – *The open loop (KG) being stable one has the property:*

$$\int_{0}^{0.5 f_S} \log\left|S_{yp}(e^{-j2\pi f/f_S})\right| df = 0$$

The sum of the areas between the curve of Syp and the axis 0dB taken with their sign is null

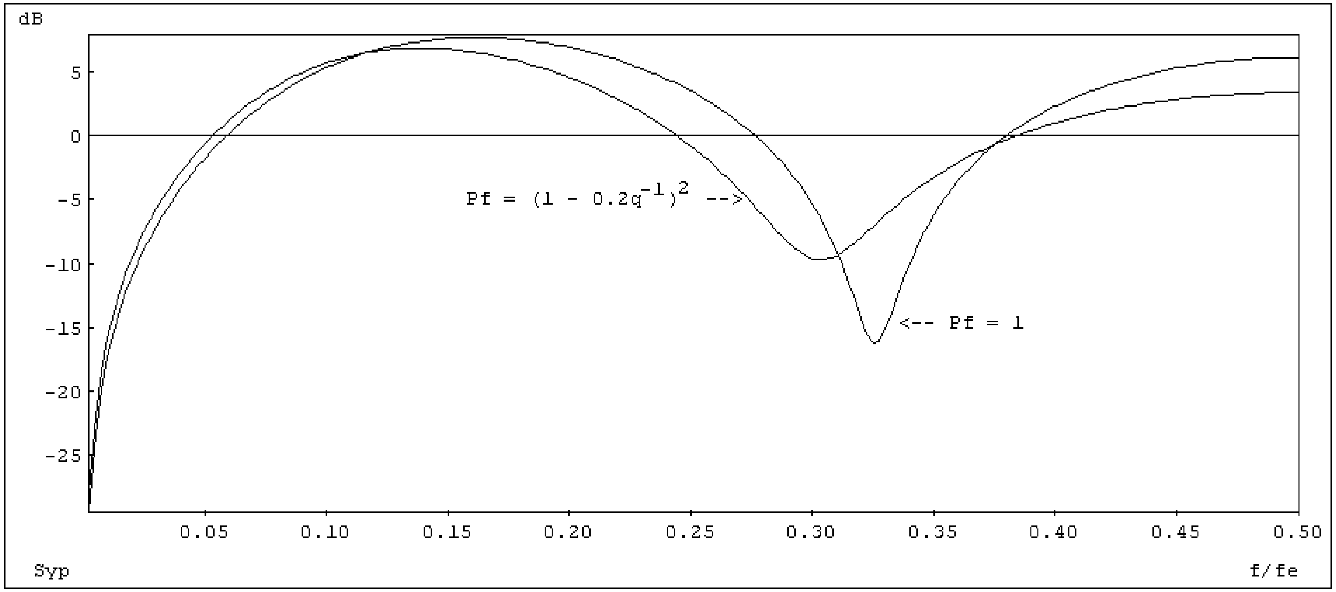
$\Downarrow$

Disturbance attenuation in a frequency region implies amplification of the disturbances in other frequency regions!

Properties of the output sensitivity function

The asymptotically stable auxiliary poles (P_F) lead in general to the reduction of $\left|S_{yp}(j\omega)\right|$ *in the frequency regions corresponding to the attenuation regions for $1/P_F$*

$$P_F(q^{-1}) = (1 + p'q^{-1})^{n_{P_F}} \qquad -0.5 \le p' \le -0.05 \qquad n_{P_F} \le n_P - n_{P_D}$$



In many applications the introduction of damped high frequency auxiliary poles is enough for assuring the required robustness margins

Properties of the output sensitivity function

Simultaneous introduction of a fixed part H_{Si} and of a pair of auxiliary poles P_{Fi} of the form:

$$\frac{H_{S_i}(q^{-1})}{P_{F_i}(q^{-1})} = \frac{1+\beta_1 q^{-1} + \beta_2 q^{-2}}{1+\alpha_1 q^{-1} + \alpha_2 q^{-2}}$$

Obtained by the discretization of :

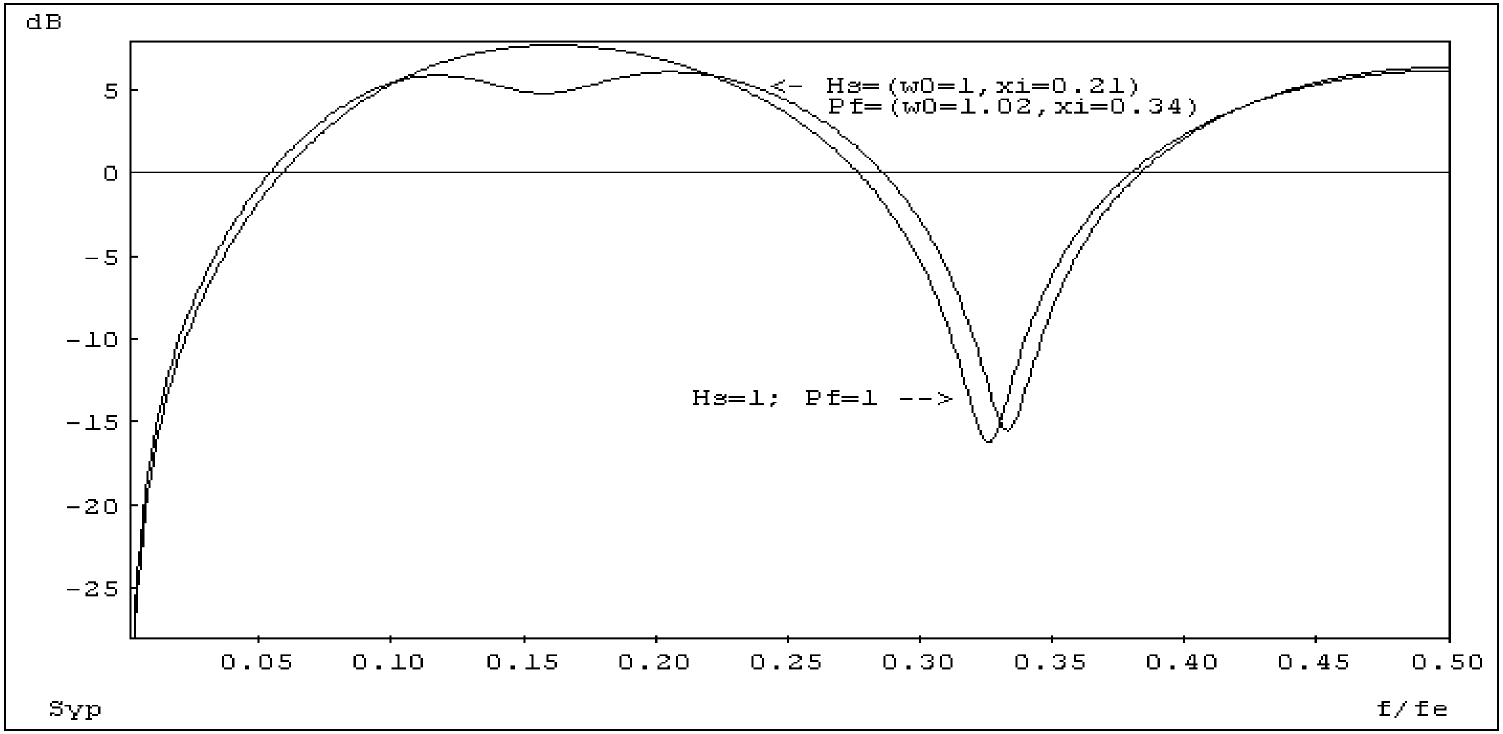
$$F(s) = \frac{s^2 + 2\zeta_{num}\omega_0 s + \omega_0^2}{s^2 + 2\zeta_{den}\omega_0 s + \omega_0^2} \quad \text{with:} \quad s = \frac{2}{T_e}\frac{1-z^{-1}}{1+z^{-1}}$$

produce and attenuation (hole) at the normalized discretized frequency:

$$\omega_{disc} = 2\arctan\left(\frac{\omega_0 T_e}{2}\right) \quad \text{with attenuation:} \quad M_t = 20\log\left(\frac{\zeta_{num}}{\zeta_{den}}\right) \quad (\zeta_{num} < \zeta_{den})$$

and has negligible effects at $f << f_{disc}$ and at $f >> f_{disc}$

Properties of the output sensitivity function



For details see Landau: Commande des Systèmes, Hermes

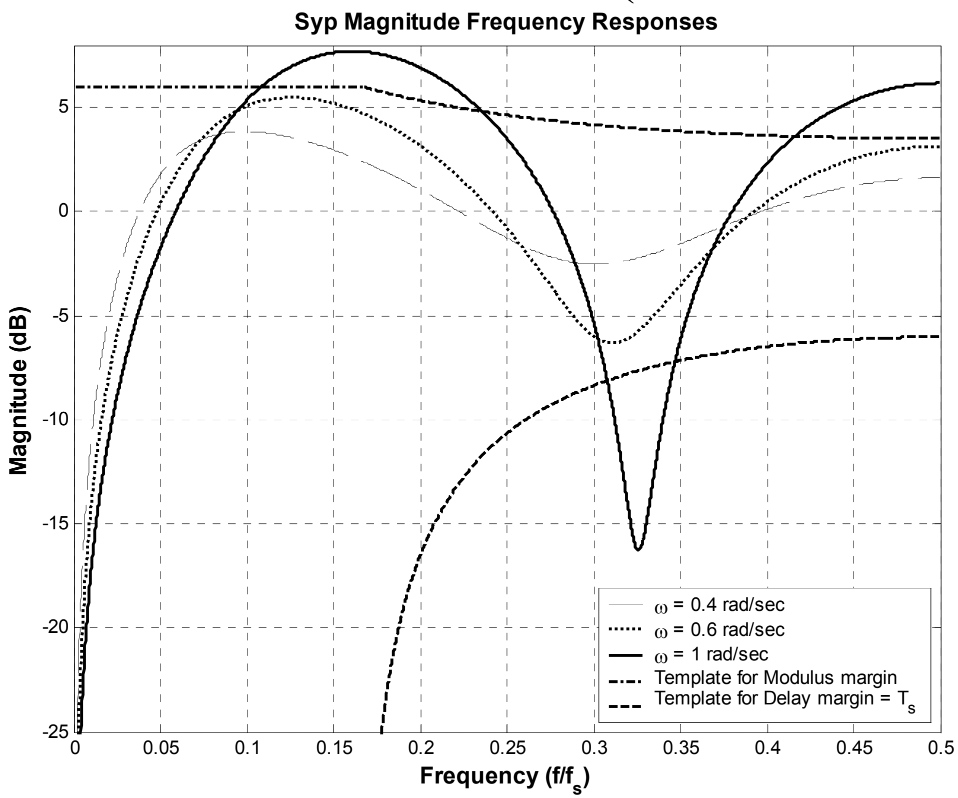
Efective computation using: *filter22.sci (.m)*

Properties of the output sensitivity function

Augmenting the attenuation or widening the attenuation zone

Higher amplification of disturbances outside the attenuation zone

Reduction of the robustness (reduction of the modulus margin)

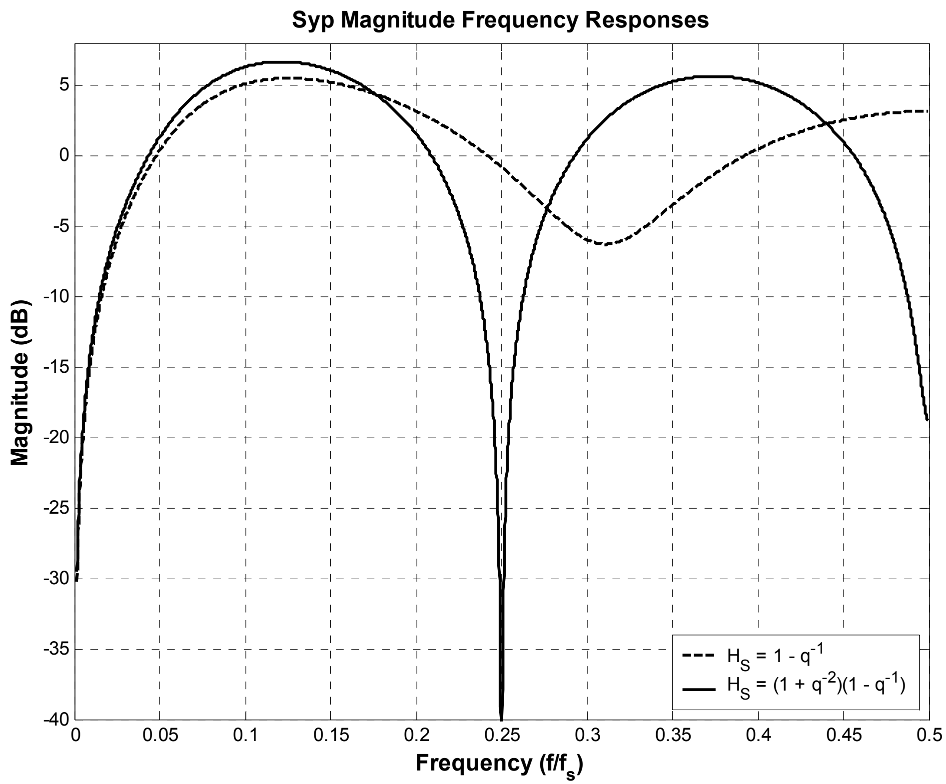


Properties of the output sensitivity function

P.4 – *Cancellation of the disturbance effect at a certain frequency:*

$$\underbrace{A(e^{-j\omega})S(e^{-j\omega})}_{\text{Zeros of } S_{yp}} = A(e^{-j\omega})H_S(e^{-j\omega})S'(e^{-j\omega}) = 0 \quad ; \quad \omega = 2\pi\, f / f_s$$

Allows introduction of zeros at desired frequencies

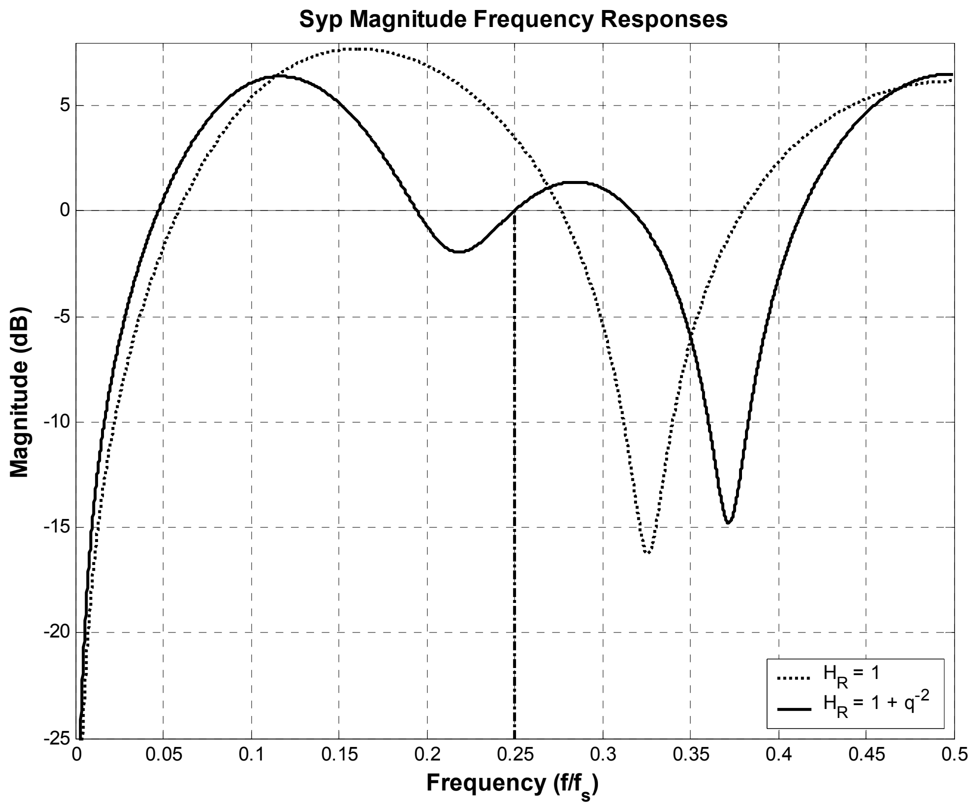


Properties of the output sensitivity function

P.5 - $\left|S_{yp}(j\omega)\right| = 1\,(0\ dB)$ *at frequencies where:*

$$B^*(e^{-j\omega})R(e^{-j\omega}) = B^*(e^{-j\omega})H_R(e^{-j\omega})R'(e^{-j\omega}) = 0\ ;\ \omega = 2\pi\, f / f_s$$

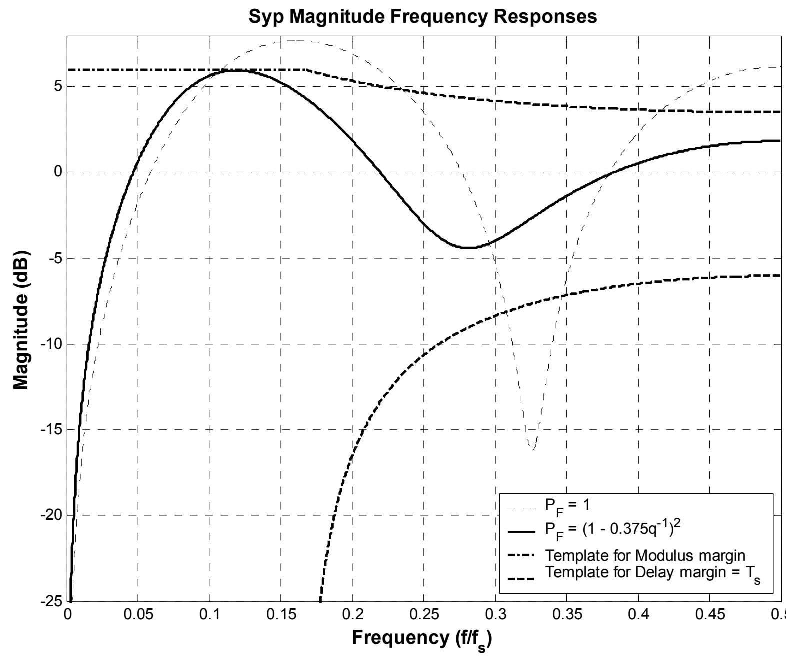
Allows introduction of zeros at desired frequencies



Properties of the output sensitivity function

P.6 – *Asymptotically stable auxiliary poles (P_F) lead (in general) to the reduction of* $\left|S_{yp}(j\omega)\right|$ *in the attenuation band of $1/P_F$*

$$P_F(q^{-1}) = (1 + p'q^{-1})^{n_{P_F}} \qquad -0.5 \le p' \le -0.05 \qquad n_{P_F} \le n_P - n_{P_D}$$



In many applications, introduction of high frequency auxiliary poles is enough for assuring the required robustness margins

Properties of the output sensitivity function

P.7 – *Simultaneous introduction of a fixed part* H_{Si} *and of a pair of auxiliary poles* P_{Fi} *having the form:*

$$\frac{H_{S_i}(q^{-1})}{P_{F_i}(q^{-1})} = \frac{1+\beta_1 q^{-1} + \beta_2 q^{-2}}{1+\alpha_1 q^{-1} + \alpha_2 q^{-2}}$$

resulting from the dicretization of :

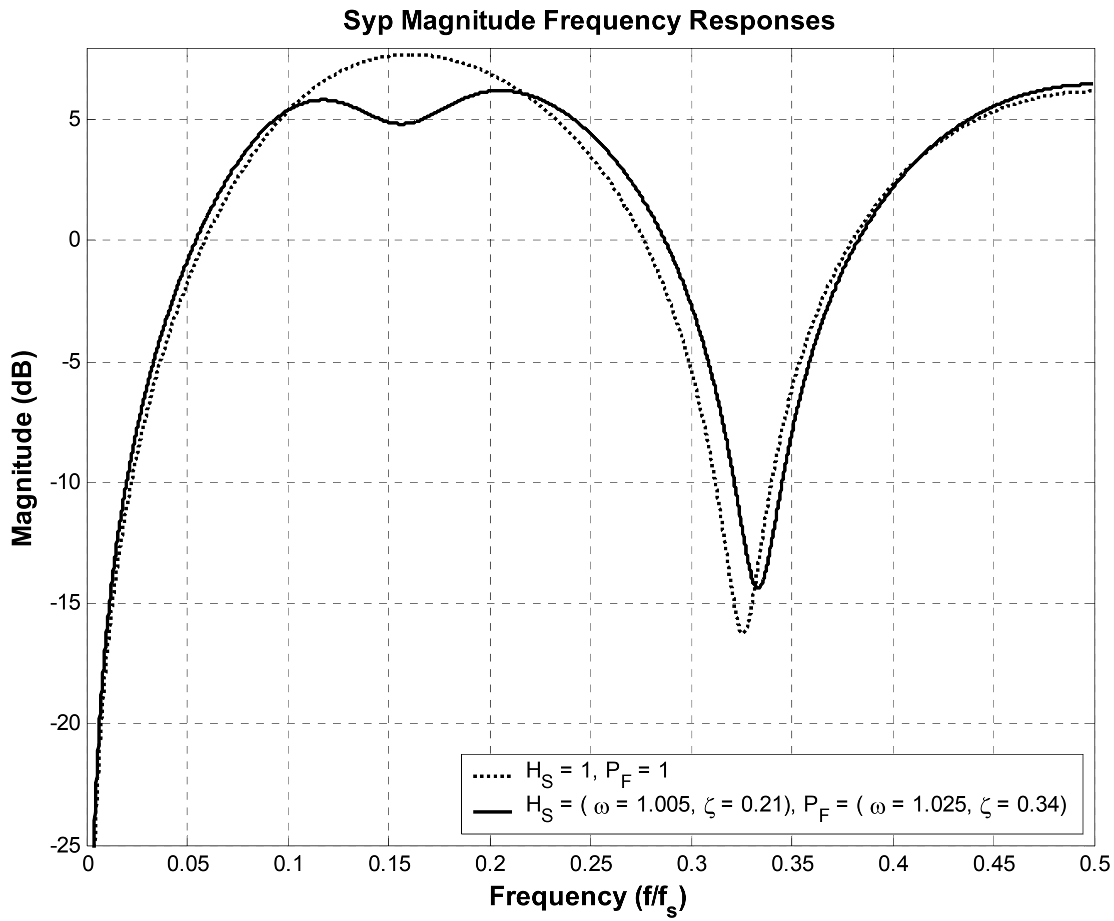
$$F(s) = \frac{s^2 + 2\zeta_{num}\omega_0 s + \omega_0^2}{s^2 + 2\zeta_{den}\omega_0 s + \omega_0^2} \qquad \textit{with:} \qquad s = \frac{2}{T_e}\frac{1-z^{-1}}{1+z^{-1}}$$

introduces an attenuation at the normalized discretized frequency:

$$\omega_{disc} = 2\arctan\left(\frac{\omega_0 T_e}{2}\right) \quad \textit{with the attenuation: } M_t = 20\log\left(\frac{\zeta_{num}}{\zeta_{den}}\right) \quad (\zeta_{num} < \zeta_{den})$$

and with negligible effect at $f << f_{disc}$ *and at* $f >> f_{disc}$

Properties of the output sensitivity function



Effective computation with the function: *filter22.sci (.m)*

Properties of the input sensitivity function

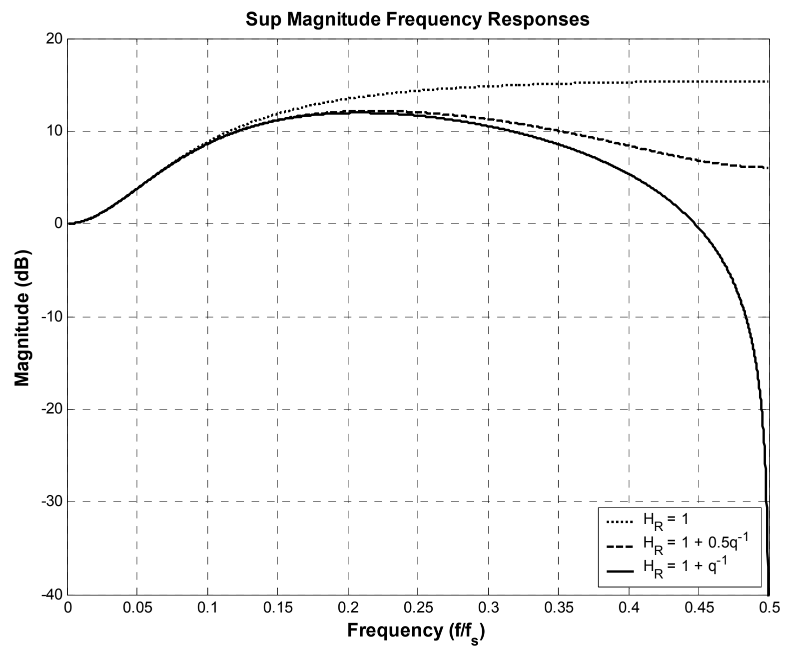
P.1 – *Cancellation of the disturbance effect on the input at a certain frequency (S_{up} = 0):*

$$A(e^{-j\omega})H_R(e^{-j\omega})R'(e^{-j\omega}) = 0 \quad ; \quad \omega = 2\pi\, f / f_s$$

Allows introduction of zeros at desired frequencies

$$H_R(q^{-1}) = 1 + \beta q^{-1} \qquad 0 < \beta \le 1$$

(active at $0.5f_S$)



Rem: The system operate in open loop at this frequency

Properties of the input sensitivity function

P.2 – *At frequencies where:*

$$A(e^{-j\omega})H_S(e^{-j\omega})S'(e^{-j\omega}) = 0 \quad ; \quad \omega = 2\pi f / f_s$$

One has:

$$\left|S_{yp}(j\omega)\right| = 0 \qquad \left|S_{up}(e^{-j\omega})\right| = \left|\frac{A(e^{-j\omega})}{B(e^{-j\omega})}\right|$$

Inverse of the system gain

Consequence : strong attenuation of the disturbances should be done only in the frequency regions where the system gain is enough large (in order to preserve robustness and avoid too much stress on the actuator)

Remember: $\left|S_{up}(j\omega)\right|^{-1}$ gives the tolerance with respect to additive uncertainties on the model (high $\left|S_{up}(j\omega)\right|$ = weak robustness)

Properties of the input sensitivity function

P.3 – *Simultaneous introduction of a fixed part* H_{Ri} *and of a pair of auxiliary poles* P_{Fi} *having the form:*

$$\frac{H_{R_i}(q^{-1})}{P_{F_i}(q^{-1})} = \frac{1+\beta_1 q^{-1}+\beta_2 q^{-2}}{1+\alpha_1 q^{-1}+\alpha_2 q^{-2}}$$

resulting from the dicretization of :

$$F(s) = \frac{s^2+2\zeta_{num}\omega_0 s+\omega_0^2}{s^2+2\zeta_{den}\omega_0 s+\omega_0^2} \qquad \text{with:} \qquad s = \frac{2}{T_s}\frac{1-z^{-1}}{1+z^{-1}}$$

introduces an attenuation at the normalized discretized frequency:

$$\omega_{disc} = 2\arctan\left(\frac{\omega_0 T_e}{2}\right) \quad \text{with the attenuation:} \; M_t = 20\log\left(\frac{\zeta_{num}}{\zeta_{den}}\right) \quad (\zeta_{num} < \zeta_{den})$$

and with negligible effect at $f << f_{disc}$ *and at* $f >> f_{disc}$

Shaping the sensitivity functions - Example I

Plant: $A = 1 - 0.7q^{-1}$; $B = 0.3q^{-1}$; $d = 2$; $T_e = 1s$

Specifications:

- Integrator
- Dominant poles: discretization of a cont. time 2nd order system : $\omega_0 = 1$ rad/s, $\zeta = 0.9$

Controller A :

Attenuation band: *0* up to *0.058 Hz* but $\Delta M < -6$ *dB and* $\Delta\tau < T_s$

Objective: same attenuation band but with $\Delta M > -6$ *dB and* $\Delta\tau > T_s$

- insertion of auxiliary poles: $P_F = \left(1 - 0.4q^{-1}\right)^2$

Controller B : good margins but reduction of the attenuation band

-insertion of pole-aero filter H_S/P_F centered at

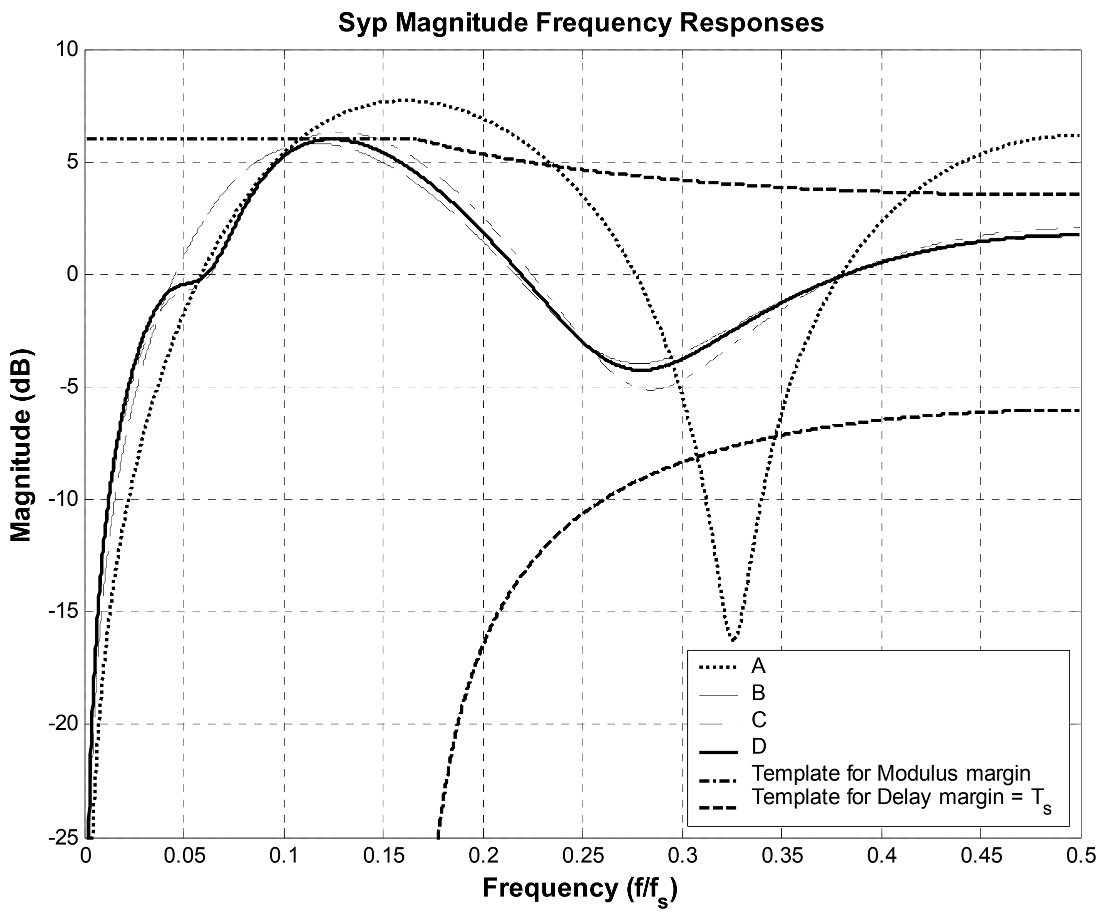
$\omega_0 = 0.4$ rad/s (0.064 Hz)

Controller C : good attenuation band but $S_{yp} > 6$ *dB*

- larger (slower) auxiliary poles (0.4 → 0.44)

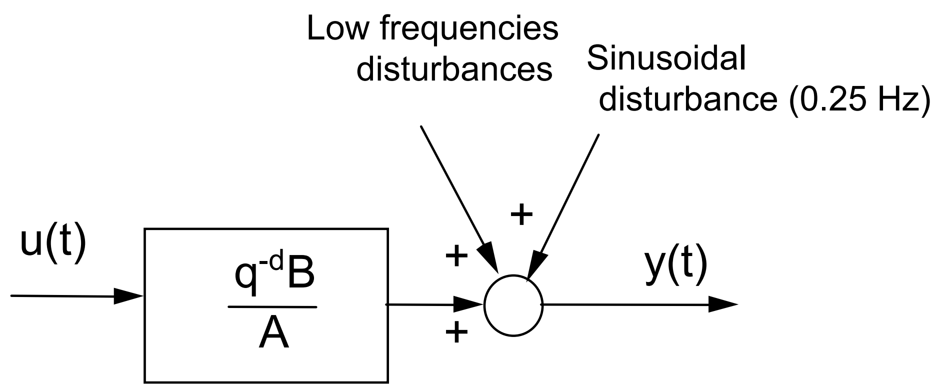
Controller D : Correct

Shaping the sensitivity functions - Example I



Shaping the sensitivity functions - Example II

Plant (integrator): $A = 1 - q^{-1}$; $B = 0.5q^{-1}$; $d = 2$; $T_s = 1s$



Specifications:

1. No attenuation of the sinusoidal disturbance at (0.25 Hz)
2. Attenuation band at low frequencies : 0 à 0.03 Hz
3. Disturbances amplification at 0.07 Hz: < 3dB
4. Modulus margin > -6 dB and Delay margin > T
5. No integrator in the controller

Shaping the sensitivity functions - Example II

- Fixed parts design : $H_R = 1 + q^{-2}$; $H_S = 1$

Opening the loop at 0.25 Hz

-Dominant poles: discretization of a cont. time 2nd order system:

$\omega_0 = 0.628$ rad/s, $\zeta = 0.9$

Controller A : the specs. at 0.07 Hz are not fulfilled

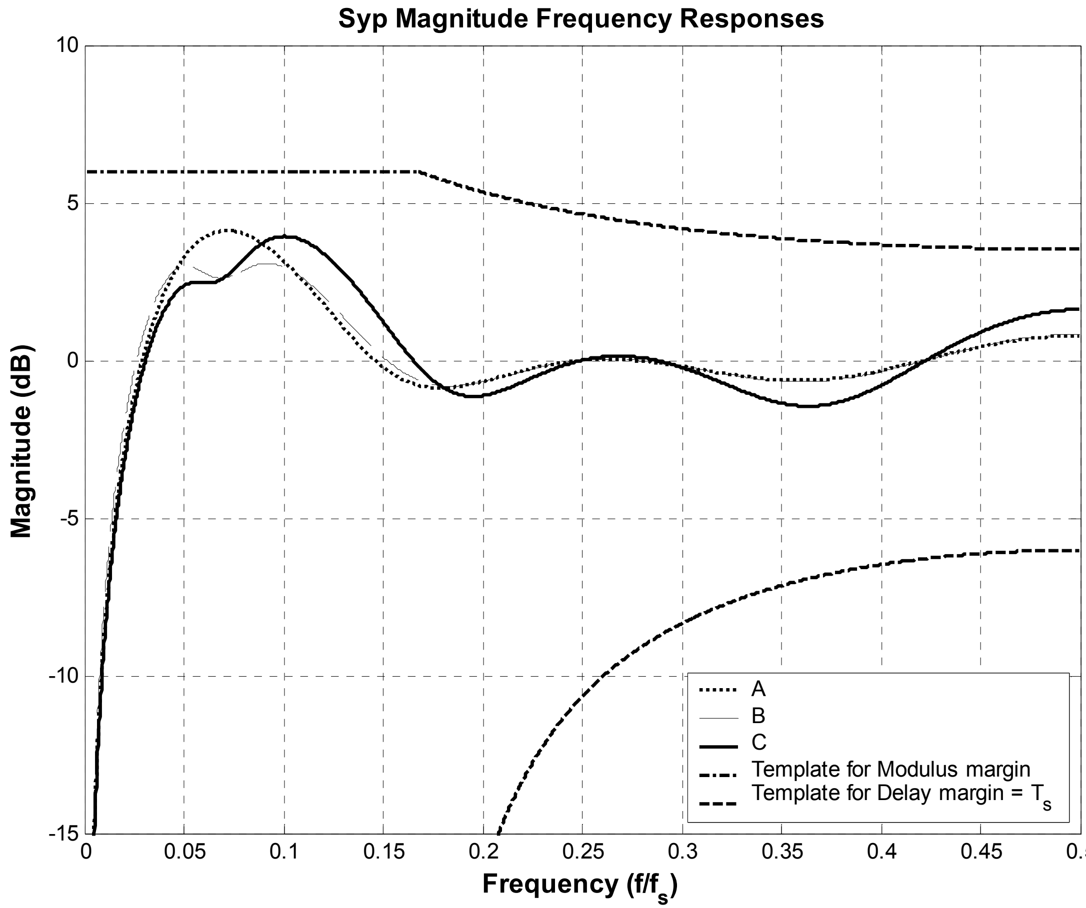
- insertion of a pole-zero filter H_S/P_F centered at $\omega_0 = 0.44$ rad/s

Controller B : Attenuation band smaller than that specified

- dominant poles acceleration: $\omega_0 = 0.9$ rad/s

Controller C : Correct

Shaping the sensitivity functions - Example II



Robust Controller Design

Pole placement with sensitivity functions shaping

Nominal performance: P_D *and part of* H_R *and* H_S

$$P = P_D P_F$$

$$R = R' H_R$$

$$S = S' H_S$$

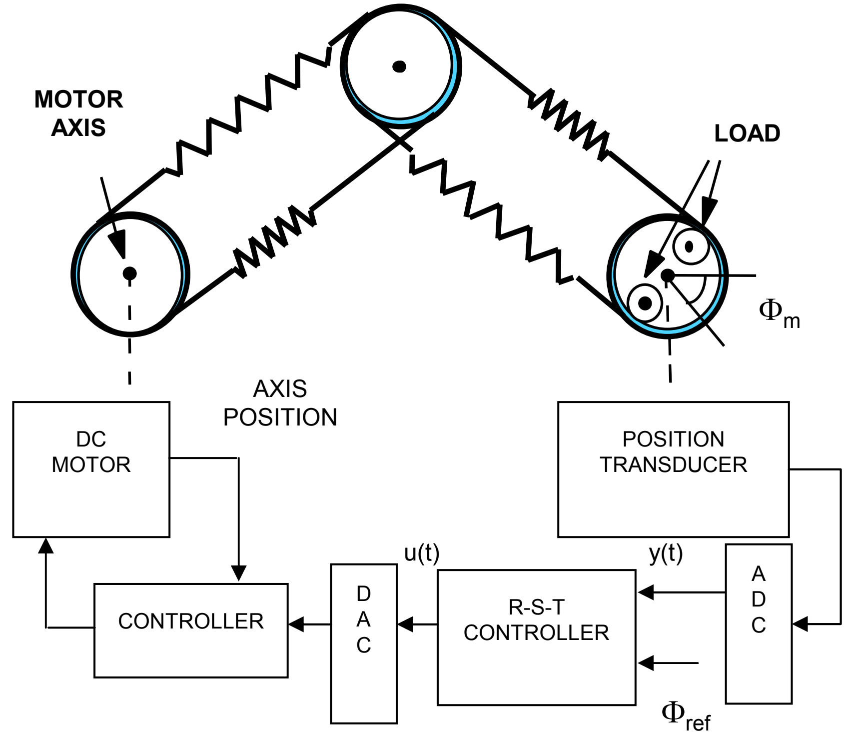
Allow to shape the sensitivity functions

Several approaches to design :

-Iterative
Choosing P_F *and using band stop filters* H_{Ri} / P_{Fi} , H_{Sj} / P_{Fj}
(matlab toolbox « ppmaster »)

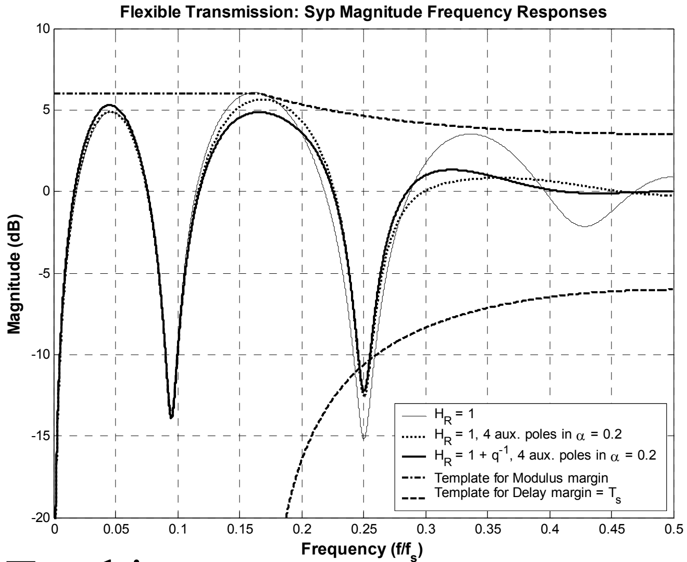
-Convex optimization
(see Langer, Landau, Automatica, June99, *Optreg* (Adaptech))

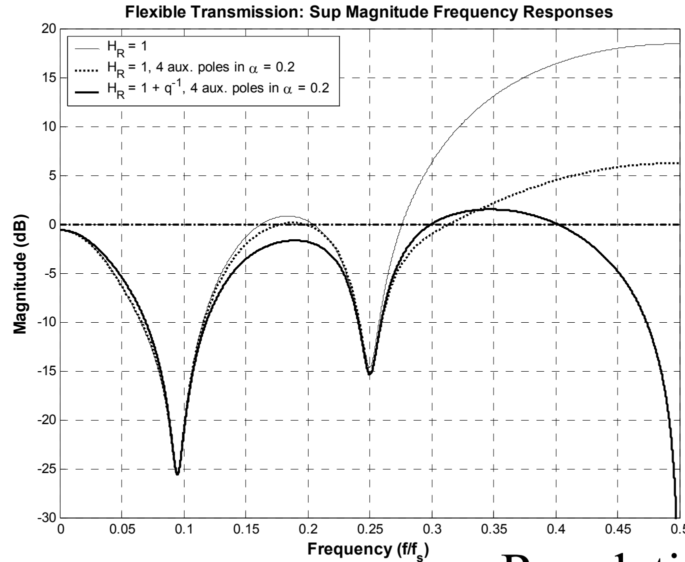
Position Control by means of a Flexible Transmission



For details see next slide and book

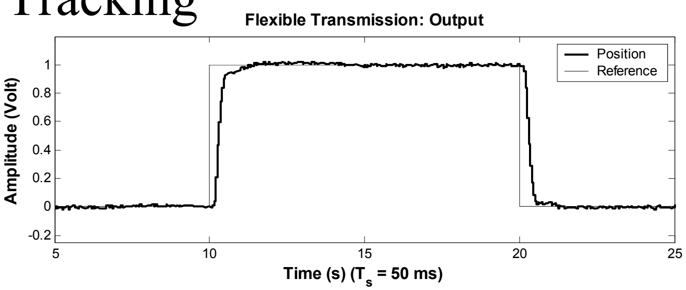
Position Control by means of a Flexible Transmission

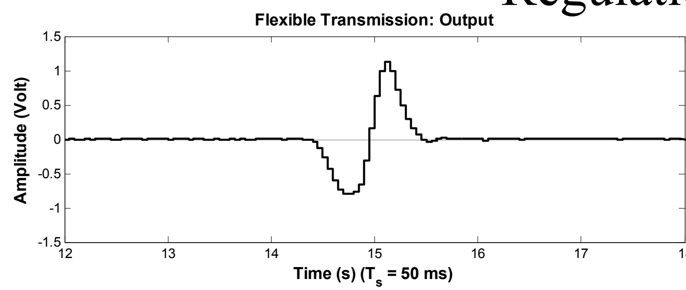


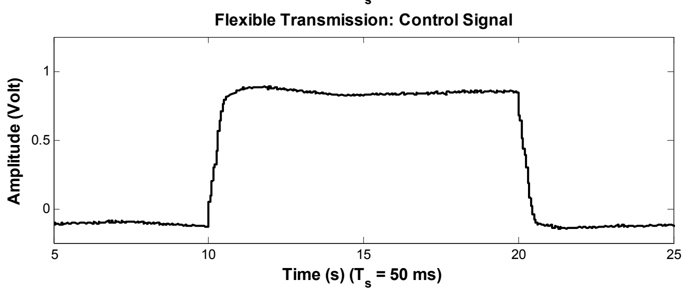


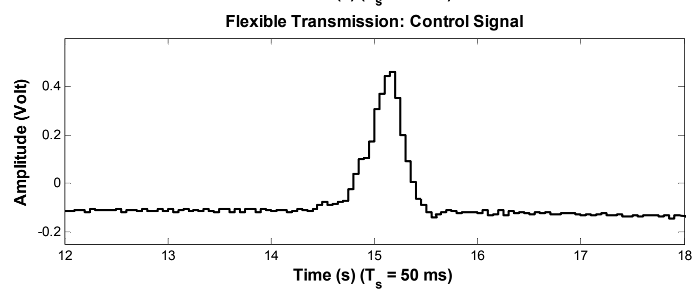
Tracking

Regulation

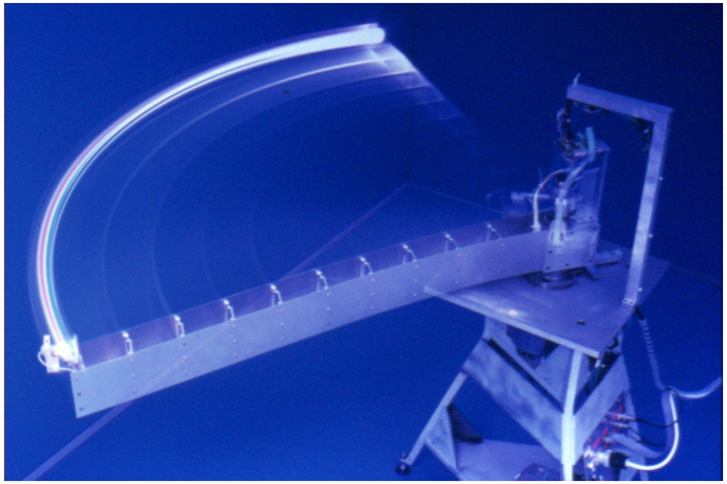


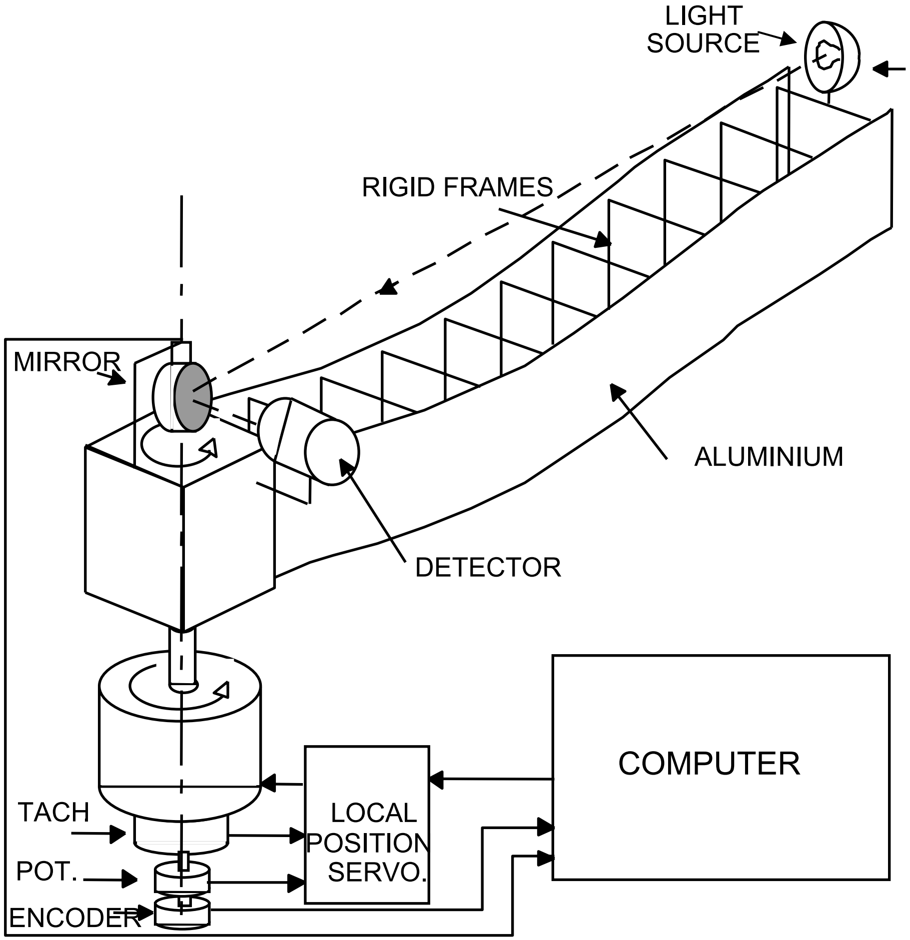




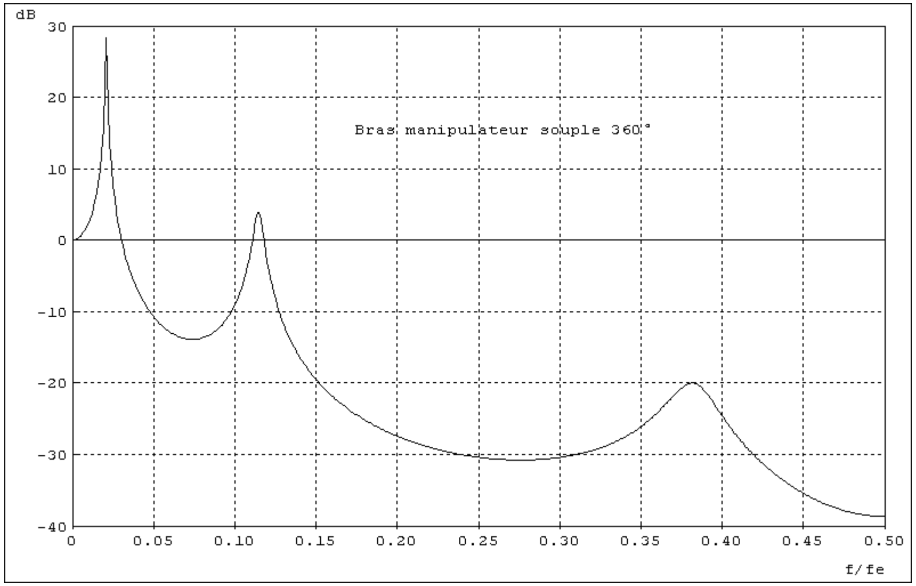


360° Flexible Arm

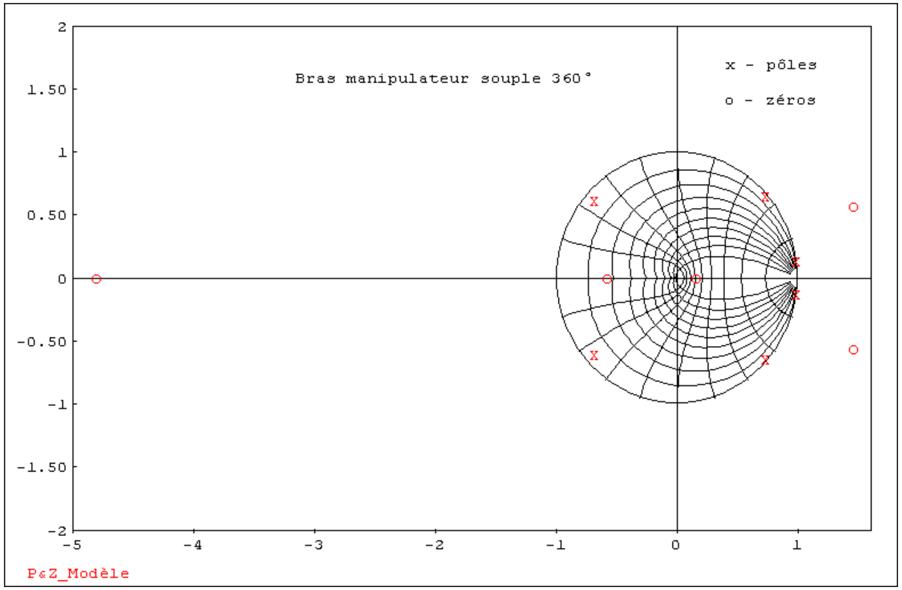




360° Flexible Arm



Frequency characteristics

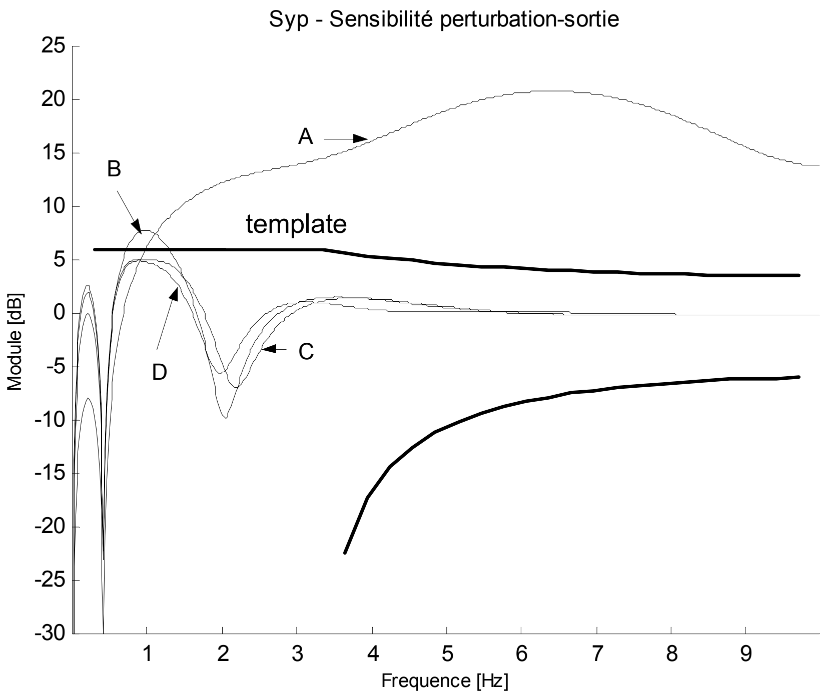


Poles-Zeros

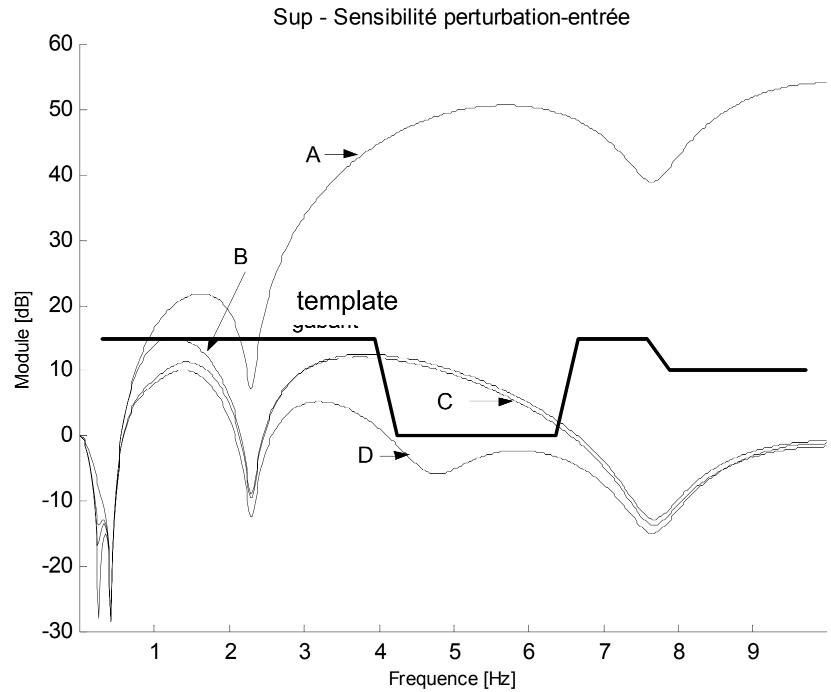
(Identified Model)

Shaping the Sensitivity Functions

Output Sensitivity Function - S_{yp}



Input Sensitivity Function - S_{up}



A- without auxiliary poles
B- with auxiliary poles
C- with stop band filter H_{S1} / P_{F1}
D- with stop band filter H_{R2} / P_{F2}